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Subject: Mathematics **Paper Name: Real Analysis-II**

Class: B.Sc.II (Semester-IV)

Question Bank

Q.	**Define the following terms.**	
	1.	Convergent sequence
	2.	Limit superior
	3.	Limit inferior
	4.	Infinite series of real number
	5.	Sequence of partial sum
	6.	Convergent series
	7.	Cauchy's principal for convergence of series
	8.	Series of positive terms
	9.	Comparison test
	10.	Limit form test
	11.	D' Alembert ratio test
	12.	Cauchy's root test
	13.	Leibnitz test
	14.	Absolutely convergent series
	15.	Conditionally convergent series
	16.	Sequence of functions
	17.	Series of functions
	18.	Limit function of sequence of functions
	19.	Alternating series
	20.	Pointwise convergence of sequence of function
	21.	Uniform convergence of sequence of function
	22.	Power series
	23.	Radius of convergence
	24.	Pointwise convergence of series of function
	25.	Uniform convergence of series of function
Q.	**Long Answer Questions.**	
	1.	If $\{s_n\}_{n=1}^{\infty}$ *and* $\{t_n\}_{n=1}^{\infty}$ are bonded sequences of real numbers then prove that i) $\overline{lim}(s_n + t_n) \leq \overline{lim}(s_n) + \overline{lim}(t_n)$ ii) $\underline{lim}(s_n + t_n) \geq \underline{lim}(s_n) + \underline{lim}(t_n)$.

	2.	Show that a positive term series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if and only if $p>1$.
	3.	Let $f_n \to f$ uniformly on an interval S. If each function f_n is continuous at a point c in S then prove that, the limit function f is also continuous at c.
	4.	If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bonded sequences of real numbers with $s_n \le t_n; \forall n$ then prove that i) $\overline{lim}\, s_n \le \overline{lim}\, t_n$ and ii) $\underline{lim}\, s_n \le \underline{lim}\, t_n$.
	5.	Show that a positive term series $\sum_{n=1}^{\infty}\frac{1}{n}$ is not convergent.
	6.	Let series of functions $\sum u_k$ converges uniformly to the function f on a set S and if each term u_k is continuous at a point c in S then prove that, the function f is also continuous at c.
	7.	Examine the convergence of sequence of function $\{x^n\}_{n=1}^{\infty}$ on $[0,1)$.
	8.	Show that, $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ is convergent by Leibnitz test.
	9.	Show that, $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ is convergent by Leibnitz test.
	10.	Show that, $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ is convergent by Leibnitz test.
	11.	If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers and $\overline{lim}\, s_n = \underline{lim}\, s_n = L$ then show that $\{s_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} s_n = L$
	12.	Prove that $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^3+n}}$ is convergent using limit form test.
	13.	Prove that $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^5+n+1}}$ is convergent using limit form test.
	14.	Examine the convergence of sequence of function $\{x^n\}_{n=1}^{\infty}$ on $(0,1)$.
	15.	
Q.	**Short Answer Questions.**	
	1.	If $\{s_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then show that $\overline{lim}(s_n) = \lim_{n\to\infty} s_n$.
	2.	If $\sum_{n=1}^{\infty} a_n$ is convergent infinite series then prove that, $\lim_{n\to\infty} a_n = 0$.
	3.	Prove that $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^3+4n+7}}$ is convergent using limit form test.
	4.	Find limit superior and limit inferior of the sequence $\{1+(-1)^n\}_{n=1}^{\infty}$
	5.	Find limit function of the sequence $\{x^n\}_{n=1}^{\infty}$ on $[0,1]$.
	6.	Prove that every absolutely convergent series is convergent.
	7.	If $\{s_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then show that $\underline{lim}(s_n) = \lim_{n\to\infty} s_n$.

	8.	Prove that, a positive term series converges if and only if its sequence of partial sum is bounded above.
	9.	Examine the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ using ratio test.
	10.	Find limit superior and limit inferior of the sequence $\{1-\frac{1}{n}\}_{n=1}^{\infty}$.
	11.	Find limit function of the sequence $\{x^n\}_{n=1}^{\infty}$on $(-1,1)$.
	12.	Find the interval of convergence for the series$\sum_{n=1}^{\infty}(2x)^n$.
	13.	Examine the convergence of the series $\sum_{n=1}^{\infty}2^n$ using Cauchy root test.
	14.	Find limit function of the sequence $\{x^n\}_{n=1}^{\infty}$on $(-1,1]$.
	15.	Find the interval of convergence for the series$\sum_{n=1}^{\infty}(x)^n$.
	16.	Find the interval of convergence for the series$\sum_{n=1}^{\infty}(3x)^n$.
	17.	Find the interval of convergence for the series$\sum_{n=1}^{\infty}\frac{(-x)^n}{n^2}$.
	18.	Find the interval of convergence for the series$\sum_{n=1}^{\infty}\frac{(-x)^n}{n^3}$.
	19.	Find the interval of convergence for the series$\sum_{n=1}^{\infty}\frac{(-x)^n}{n}$.
	20.	Find limit superior and limit inferior of the sequence $\{\frac{1}{n}\}_{n=1}^{\infty}$.
	21.	Find limit superior and limit inferior of the sequence $\{(-1)^n\}_{n=1}^{\infty}$.
	22.	Find limit superior and limit inferior of the sequence $\{1,2,1,2,1,\ldots\}$.
	23.	Find limit function of the sequence $\{x^n\}_{n=1}^{\infty}$on $(-1,1)$.
	24.	Find the radius of convergence of $\sum_{n=1}^{\infty}\frac{(-x)^n}{n^2}$.
	25.	Find the radius of convergence of $\sum_{n=1}^{\infty}\frac{(-x)^n}{n}$.
	26.	Find the radius of convergence of $\sum_{n=1}^{\infty}(x)^n$.
	27.	Find the radius of convergence of $\sum_{n=1}^{\infty}(2x)^n$.
	28.	Find the radius of convergence of $\sum_{n=1}^{\infty}(3x)^n$.
	29.	Find the region of convergence of $\sum_{n=1}^{\infty}(2x)^n$.
	30.	Find the region of convergence of $\sum_{n=1}^{\infty}(3x)^n$.

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1: Answer in one sentence

1) Cyclic group

2) Index of a subgroup

3) Quotient group

4) Euler's $\emptyset$ function

5) Cosets of subgroup

6) Simple group

7) Order of an element of a group

8) Kernel of a homomorphism

9) Congruence relation

10) Centre of a group

11) Subgroup

12) Normalizer of an element $'a'$ of a group

13) Generator of cyclic group

14) Normal subgroup

15) Proper normal subgroup

16) Homomorphism of groups

17) Isomorphism

18) Endomorphism

19) Monomorphism

20) Group

21) Abelian group

22) Permutation group

23) Automorphism

24) Transposition

25) Equivalence class

2. Long answer questions

1) Show that if G is a finite group and H is a subgroup of G then

   $o(H)$ divides $o(G)$.

2) Prove that for any integer a and prime $p > 0$ then $a^p \equiv a(modp)$.

   Find the remainder of 3^{47} when divided by 23.

3) If $f: G \to G'$ is a homomorphism. Show that

   $i)$ $f(e) = e'$.

   $ii)$ $f(x^{-1}) = [f(x)]^{-1}$.

   $iii)$ $f(x^n) = [f(x)]^n$, n is an integer.

   Where e, e' are identity elements of $G,\ G'$ respectively.

4) Show that a non empty subset H of a group G is a subgroup of G if and only if

   $i)$ $a,\ b \in H \Rightarrow ab \in H$

   $ii)$ $a \in H \Rightarrow a^{-1} \in H$

5) Prove that order of a cyclic group is equal to the order of its generator.

6) If mapping $f: G \to G'$ be an onto homomorphism with $K = kerf$ then show that $\frac{G}{K} \cong G'$.

7) Show that a non empty subset H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$.

8) Prove that subgroup of a cyclic group is cyclic.

9) Show that a subgroup H of a group G is normal in G if and only if $g^{-1}hg \in H$ for all $h \in H,\ g \in G$.

10) Let H and K be two subgroups of group G, where H is normal in G

then show that $\frac{HK}{H} = \frac{K}{H \cap K}$.

11) If H and K are two normal subgroups of group G such that $H \subseteq K$

then show that $\frac{G}{K} \cong \frac{G/H}{K/H}$.

12) Show that every group G is isomorphic to a permutation group.

13) Show that a subgroup H of a group G is a normal subgroup of G if and only if product of two right cosets of H in G is again a right coset of H in G.

14) Prove that for any integer a and prime $p > 0$ then $a^p \equiv a(mod p)$.

Find the remainder of 4^{107} when divided by 13.

15) State and prove Lagrange's theorem.

3. Short answer questions

1) Show that if H_1 and H_2 are two subgroups of a group G then $H_1 \cap H_2$ is also a subgroup of G.

2) Show that an infinite cyclic group has precisely two generators.

3) Show that a subgroup H of a group G is normal in G if and only if $g^{-1}Hg = H$ for all $g \in G$.

4) Show that the intersection of any two normal subgroups of a group is a normal subgroup.

5) Show that a homomorphism $f: G \rightarrow G'$ is one-one if and only if $ker f = \{e\}$.

6) For a finite group G, show that order of any element of G divides order of G.

7) Let H be a subgroup of G. Show that $Ha = H$ if and only if $a \in H$.

8) Show that centre of a group G is a subgroup G.

9) By using Fermat's theorem find the remainder of 8^{103} when divided by 13.

10) Show that every subgroup of an abelian group is normal.

11) By using Fermat's theorem find the remainder of 4^{107} when divided by 13.

12) Show that every quotient group of a cyclic group is a cyclic.

13) Let $< Z, + >$ and $< E, + >$ be the groups of integers and even integers. Define mapping $f: Z \rightarrow E$ such that $f(x) = 2x$ for all $x \in Z$. Show that f is isomorphic.

14) Show that normalizer of $a \in G$ is subgroup of G.

15) Let H be a subgroup of G. Show that $Ha = Hb$ if and only if $ab^{-1} \in H$.

16) By using Fermat's theorem find the remainder of 3^{47} when divided by 23.

17) Prove that for any integer a and prime $p > 0$ then $a^p \equiv a(modp)$.

18) By using Euler's theorem, find the remainder of 2^{48} when divided by 105.

19) Show that HK is a subgroup of G if and only if $HK = KH$.

20) Show that $Ha = \{x \in G | \, x \equiv a(modH)\}$ for any $a \in G$.

21) Show that Ha is a subgroup of G if and only if $a \in H$.

22) Show that centre of a group G is a subgroup of G.

23) Let a be an element of group G. Show that the set H of all integral powers of $'a'$ is a subgroup of G.

24) Show that every quotient group of an abelian group is abelian.

25) Show that every quotient group of a cyclic group is a cyclic.

26) G is finite group and N is a normal subgroup of $'G'$ then show that $o\left(\frac{G}{N}\right) = \frac{o(G)}{o(N)}$.

27) Let N be a normal subgroup of a group then show that $\frac{o(Na)}{o(a)}$ for any $a \in G$.

28) Show that any infinite cyclic group is isomorphic to the group of integers.

29) Suppose G is a group and N is a normal subgroup of G. Let $f: G \rightarrow \frac{G}{N}$ defined by $f(x) = Nx$, for $x \in G$. Show that f is homomorphism of G onto $\frac{G}{N}$.

30)Show that the mapping $f: Z \rightarrow Z$ such that $f(x) = -x$ for all $x \in Z$ is an automorphism of the additive group of integers z.