

Rayat Shikshan Sanstha's
Yashavantrao Chavan Institute of Science, Satara (Autonomous)

Department of Mathematics

Academic Year 2021-22

Subject: Mathematics

Paper Name: Discrete Mathematics

Class: B.Sc.III (Semester-VI)

Question Bank

Q. Define the following terms.

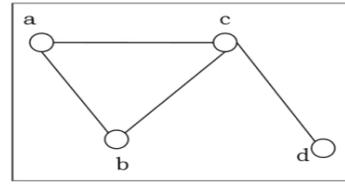
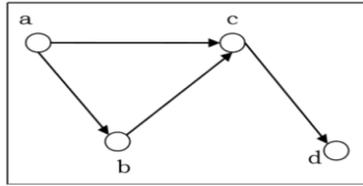
1. Compound statement
2. Logic
3. Logical equivalence
4. Tautology
5. Contradiction
6. Negation
7. Contrapositive of conditional statement
8. Inverse of conditional statement
9. Binary numbers
10. Decimal numbers
11. Octal numbers
12. Hexadecimal numbers
13. Graph
14. Simple graph
15. Loop
16. Complete graph
17. Bipartite graph
18. Complete bipartite graph
19. Subgraph
20. Walk
21. Trail
22. Path
23. Circuit
24. Degree of vertex
25. Regular graph

Q. Long Answer Questions.

1. Check whether following argument is valid or invalid?

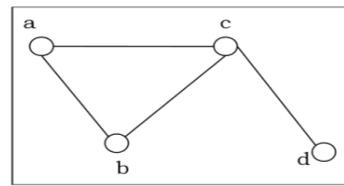
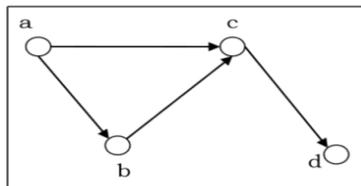
$$p \rightarrow q \vee \sim r; \quad q \rightarrow p \wedge r; \quad \therefore p \rightarrow r.$$

2. Find matrix representation of following graphs.



3. Verify logical equivalence of following statement form,

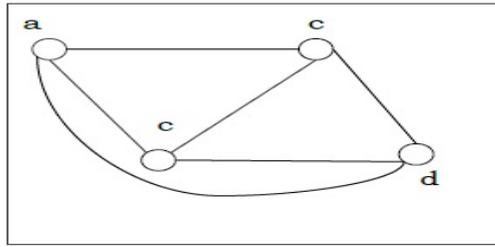
$$(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) \equiv p.$$
4. Write a rule of inferences and show that all are the valid arguments.
5. Prove that in any graph there are an even number of vertices of odd degree.
6. Define inverse and converse of conditional statements and construct their truth tables.
7. Define converse conditional statement. Show that there is no equivalence between converse of conditional statement and conditional statement.
8. Find the degree of each vertex and degree of graph of following graphs:



9. Draw all simple graphs with the four vertices $\{u, v, w, x\}$ and two edges, one of which is $\{u, v\}$.
10. Draw the complete graphs K_1, K_2, K_3, K_4 .
11. List all the subgraphs of the graph G with vertex set $\{u, v\}$ and edge set $\{e_1, e_2, e_3\}$, where the endpoints of e_1 are u and v , the endpoints of e_2 are u and v and the endpoints of e_3 is a loop at u .
12. Draw the complete bipartite graphs $K_{1,1}, K_{2,2}, K_{3,2}$
13. Prove that, the total degree of graph is twice of number of edges.
14. Verify logical equivalence pf $(p \wedge \sim q) \vee p \equiv p$
15. Construct the truth table for $(p \wedge \sim r) \leftrightarrow (q \vee r)$.

Q. Short Answer Questions.

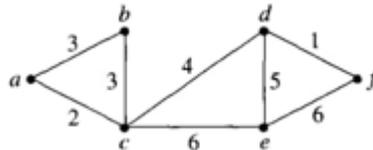
1. Show that the statement form $p \wedge \sim p$ is a contradiction.
2. Convert 4576_8 to decimal notation.
3. Convert 20763_8 to decimal notation.
4. Convert 20763_8 to binary notation.
5. Convert 4576_8 to binary notation.
6. Find the degree of each vertex of a graph and degree of graph.



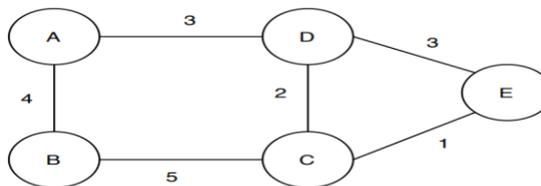
7. Find directed graph from following matrix.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

8. Write a contradiction rule and show that it is valid argument.
 9. Construct the truth table for $(p \wedge \sim r) \leftrightarrow (q \vee r)$.
 10. Construct the truth table of $p \wedge q$ and $q \vee r$
 11. Find all minimum spanning trees for the following graph.



12. Find all minimum spanning trees for the following graph.



13. Show that $p \rightarrow q \equiv \sim p \vee q$.
 14. Convert $(1101110)_2$ into octal and $(101101)_2$ into hexadecimal.
 15. Prove that, if G is any graph then sum of degrees of all the vertices of G equals twice of the number of edges of G .
 16. Write Modus Ponens form and check whether it is valid or invalid argument form.
 17. Construct the truth table for conditional statement.
 18. Construct the truth table for biconditional statement.
 19. Add 1101_2 and 111_2 using binary notation.
 20. Add 0100_2 and 0010_2 using binary notation.
 21. Subtract 1011_2 from 11000_2 using binary notation.
 22. Subtract 0010_2 from 1011_2 using binary notation.
 23. Determine whether certain graph exist or nor?
 A graph with four vertices of degree 1,1,2 and 3.
 24. Determine whether certain graph exist or nor?

- A graph with four vertices of degree 1,1,3 and 3.
- 25.** Determine whether certain graph exist or nor?
A simple graph with four vertices of degree 1,1,2 and 3.
- 26.** Prove that, the total degree of graph is even.
- 27.** Convert 2013_5 into decimal notation.
- 28.** Convert $C50A_{16}$ into binary notation
- 29.** Write the rule of inference.
- 30.** Construct the truth table for $(p \rightarrow r) \leftrightarrow (q \rightarrow r)$.

Rayat Shikshan Sanstha's
Yashavantrao Chavan Institute of Science, Satara (Autonomous)
Department of Mathematics
B.Sc.-III(Semester-VI) MATHEMATICS
Complex Analysis (BMT 603)
Subject Code: 16012

Que 1. Answer the following.

- 1) Write Cauchy- Riemann equations in cartesian form.
- 2) Define singular point
- 3) How to calculate the singularity of function $f(z)$ at infinity?
- 4) Define pole.
- 5) What is the isolated singularity?
- 6) Define the Jordan curve.
- 7) Define complex valued function.
- 8) Define meromorphic function
- 9) Define limit of a complex valued function
- 10) Write Cauchy- Riemann equations in polar form.
- 11) Define principal argument of complex number
- 12) Define harmonic function
- 13) What is the modulus and argument of $z = -i$.
- 14) Define removable singularity.
- 15) Define regular point.
- 16) Define simple curve.
- 17) Write the solution of exact differential equation $Mdx + Ndy = 0$.
- 18) Define entire function.
- 19) Write the equation of circle whose center is at c and radius a in complex plane.
- 20) Define analytic function.
- 21) Write statement of Greens theorem.
- 22) Define essential singularity.
- 23) Define cross cut.
- 24) Define smooth curve.

25) Define residue of an isolated singularity.

Que. 2 Solve the following questions.

1) Show that $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin.

2) If $f(z) = u + iv$ is an analytic function. If $f'(z) = 0$ then show that $f(z)$ is constant.

3) If $f(z) = u + iv$ is an analytic function then show that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \partial \bar{z}}$.

4) If $u = e^x \cos y$ is harmonic function then find the harmonic conjugate of u by using exact differential equation method and hence construct the analytic function.

5) Evaluate $\int \bar{z} dz$ along the line from $z = 0$ to $z = 2i$ and then from $z = 2i$ to $z = 4 + 2i$.

6) Find the value of integral $\int_0^{1+i} (x - y + ix^2) dz$ along the straight-line from $z = 0$ to $z = 1 + i$.

7) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series for the region $|z| > 3$.

8) Show that $\int_C \bar{z} dz$ along a semi-circular path from $z = -a$ to $z = a$ lies above the x axis is $-\pi ia^2$.

9) If $f(z) = \frac{z-4}{(z-3)(z-5)^2}$ then find the residues at corresponding poles.

10) Prove that analytic function with constant modulus is constant.

11) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.

12) Show that the real and imaginary part of $f(z) = e^z$ are harmonic.

13) Evaluate $\int_0^{2\pi} \frac{1+2 \cos \theta}{5+4 \cos \theta} d\theta$.

14) If $f(z) = u + iv$ is an analytic function then show that $u(x, y) = c_1$ and $v(x, y) = c_2$ represent orthogonal family of curves.

15) Evaluate $\int_0^{1+i} z^2 dz$.

16) If $f(z) = u + iv$ is an analytic function then find $f(z)$ in terms of z where $u - v = (x - y)(x^2 + 4xy + y^2)$.

- 17) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series for the region $1 < |z| < 3$.
- 18) If $f(z) = \frac{z^4}{z^2+a^2}$ then find the residues at corresponding poles.
- 19) Show that $\int_C \bar{z} dz$ along a semi-circular path from $z = -a$ to $z = a$ lies below the x axis is πia^2 .
- 20) Find the type of singularities of $f(z) = \frac{\cot\pi z}{(z-a)^2}$ at $z = a$ and $z = \infty$.
- 21) If $u = \log(x^2 + y^2)$ then construct the analytic function.
- 22) Evaluate $\int_C \frac{z^4}{z-3i} dz$ by using Cauchy integral formula along the curve $|z - 2| < 5$.
- 23) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.
- 24) What kind of singularities exist for the function $f(z) = \frac{1-e^z}{1+e^z}$ at $z = \infty$.
- 25) Find the value of integral $\int_0^{1+i} (x - y + ix^2) dz$ along the straight-line from $z = 0$ to $z = 1 + i$.
- 26) Find the residue of $f(z) = \frac{z^2}{(z^2+1)^2}$ at $z = \infty$.
- 27) Expand $f(z) = \frac{1}{z(z^2-3z+2)}$ in a Laurent's series for the region $0 < |z| < 1$.
- 28) If $f(z) = \frac{z-4}{(z-3)(z-5)^2}$ then find the residues at corresponding poles.
- 29) Evaluate $\int_C \frac{e^{az}}{z+1} dz$ over the circle $C: |z| = 2$.
- 30) Find the type of singularities of $f(z) = \tan\left(\frac{1}{z}\right)$ at $z = 0$.

Que. 3 Solve the following questions. (long answers)

- 1) State and prove necessary conditions for $f(z)$ to be analytic.
- 2) If $f(z) = u(x, y) + iv(x, y)$ is analytic function and $z = re^{i\theta}$ is a polar form of z then show that Cauchy- Riemann equation in polar form are $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$
- 3) If $f(z)$ is analytic in a simply connected domain D except at finite number of poles z_1, z_2, \dots, z_n within the closed contour and continuous on boundary which is a

rectifiable Jordan curve then prove that $\int_C f(z)dz = 2\pi i \sum_{k=1}^n \text{Res}(z = z_k)$ hence evaluate $\int_C \frac{dz}{z^3(z+3)}$ over the circle $C: |z| = 1$.

- 4) Explain exact differential equation method for construction of an analytic function. Hence construct analytic function for $u = e^x \cos y$
- 5) If $f(z)$ and $g(z)$ are analytic inside and on a simple closed curve C , if $|g(z)| < |f(z)|$ on C then show that $f(z)$ and $f(z) + g(z)$ both have same number of zeros inside C .
- 6) Explain Milne-Thomson Method for construction of an analytic function by considering both cases.
- 7) If $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$, $z \neq 0$ and $f(z) = 0, z = 0$ then show that $f(z)$ is continuous and satisfy Cauchy Riemann equations at origin and $f'(0)$ does not exist.
- 8) If $f(z)$ is analytic in a simply connected domain D except at finite number of poles z_1, z_2, \dots, z_n within the closed contour and continuous on boundary which is a rectifiable Jordan curve then prove that $\int_C f(z)dz = 2\pi i \sum_{k=1}^n \text{Res}(z = z_k)$ hence evaluate $\int_C \frac{e^{az}}{z+1} dz$ over the circle $C: |z| = 2$.
- 9) State and prove Cauchy theorem for simply connected domain.
- 10) State and prove sufficient conditions for $f(z)$ to be analytic.
- 11) If a domain D is bounded by system of closed rectifiable curves C_1, C_2, \dots, C_k and $f(z)$ is analytic in domain D and continuous on C_1, C_2, \dots, C_k then show that $\int_{C_1} f(z)dz + \int_{C_2} f(z)dz + \dots + \int_{C_k} f(z)dz = 0$
- 12) If $f(z)$ is analytic in domain D then show that $f(z)$ has derivatives of all orders at any point $z = a$ and all of which are analytic in domain D , their values are given by $f^n(a) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}} dz$ where C is any closed curve surrounding the point $z = a$.
- 13) State and prove Cauchy integral formula for simply connected domain.
- 14) Explain the method to evaluate the integral of the type $\int_{-\infty}^{\infty} f(z)dz$ hence show that $\int_0^{\infty} \frac{dz}{1+z^2} = \frac{\pi}{2}$.
- 15) State and prove Cauchy Residue theorem.

Mathematics

Entrepreneurship Development in Mathematics (SECCMT 607)

Subject Code: 16014

Q.1: Define following terms.

- 1) Innovative Entrepreneur.
- 2) Joint Stock Company.
- 3) Market Assessment.
- 4) Institutional Entrepreneur.
- 5) Cooperative Organization.
- 6) E-Commerce
- 7) Individual Entrepreneur.
- 8) Single Ownership.
- 9) Liberalization.
- 10) Partnership
- 11) General Partnership
- 12) Limited Partnership
- 13) Public Sector
- 14) Private Sector
- 15) Marketing Plan

Q.2: Long Answer Questions

- a) Explain Business reforms in India.
- b) Write a note on E- commerce.
- c) Explain growth of Entrepreneurship in India.

- d) Differentiate between Joint Stock Company and Partnership.
- e) Write a note on Market Assessment.
- f) Explain contents of a project report briefly.
- g) Explain the concept of Entrepreneur.
- h) Discuss the characteristics of an entrepreneur.
- i) Discuss types of Entrepreneurs.
- j) Classify Entrepreneurs on nature basis.

Q.3: Short Answer Questions

- i) Write a short note on Entrepreneur as a Risk Bearer.
- ii) Write four characteristics of an entrepreneur.
- iii) Explain the difference between Entrepreneur and Entrepreneurship.
- iv) Write a short note on Entrepreneur as innovator.
- v) Write any four objectives of Liberalization in India.
- vi) Explain any four types of ownership.
- vii) Write a short note on Women Entrepreneurship.
- viii) Explain the steps in transformation of an idea into an opportunity.
- ix) Explain any four objectives of Public Sector.
- x) Write a short note on Entrepreneur as organizer.
- xi) Write a short note on Entrepreneur as an innovator.
- xii) Write a short note on Technical Entrepreneur.
- xiii) Write a short note on Drone Entrepreneur.
- xiv) Write a short note on Fabien Entrepreneur.
- xv) Classify Entrepreneur on size basis.