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Class: M.Sc.I (Semester-II)

Question Bank

Q. Define the following terms.

1. Topology on set X
2. Co-countable topology
3. Co-finite topology
4. Usual topology
5. Limit point of set in topological space
6. Closure of a set in topological space
7. Closed set in topological space
8. Open set in topological space
9. Subspace of topological space
10. Continuous function
11. Open mapping
12. Closed mapping
13. Homeomorphism
14. Compact space
15. Countably compact space
16. Connected space
17. Base for topology
18. Lindelof space
19. Regular space
20. T_0 −space
21. T_1 −space
22. T_2 −space
23. Normal space
24. First axiom space
25. Second axiom space

Q. Fill in the blanks.

1. Arbitrary intersection of topologies on set X is….
2. In indiscrete topological space (X,τ),$d(A) =..; A \subseteq X\ with\ |A| = 1$
3. Closed subspace of normal space is…
4. Let, $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then $d(\{a, b\}) = \cdots$
5. A subset E of real line is connected if E is an ….
6. If X is finite set then co-finite topology on X coincides with …topology on X
7. Let (X,τ) be a topological space and let $A, B \subseteq X$ with $d(A \cup B) = \cdots$.

8. In usual topological space, derived set of $[p, r)$ is….
9. Every closed subset of compact topological space is….
10. Every closed continuous image of normal space is….
11. A set A is closed set in topological space (X, τ) iff $\bar{A} = \cdots$.
12. $i(A)$ is …. open set contained in A.
13. A subset of topological space is compact if every open cover of set has ….
14. Any compact, regular space is…
15. Every compact, Hausdorff space is $\ldots - space$
16. Let, $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}\}$ be topology on X. Then $d(\{b\}) = \cdots$
17. Let, $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}\}$ be topology on X. Then $d(\{a\}) = \cdots$
18. Let (X, τ) and (X^*, τ^*) be any topological spaces. $f: X \rightarrow X^*$ is closed if …
19. Let, $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}\}$ be topology on X. Then $d(\{c\}) = \cdots$
20. If $x\ and\ y$ are distinct points of X then there exist an open set which contain one of them but the other is called …. -space.
21. A topological space (X, τ) is T_1 −space if and only if $\{x\}$ is … in X for each $x \in X$.
22. A subset of usual topological space is compact if and only if ….
23. $d(\mathbb{N}) = \cdots$ in indiscrete topological space
24. Let (X, τ) and (X^*, τ^*) be any topological spaces. $f: X \rightarrow X^*$ is open if …
25. Let, (X, τ) be a topological space and $A, B \subseteq X$. Then $i(A \cap B) = \cdots$.

Q. Long Answer Questions.

1. If X be an infinite set and $\tau = \{\emptyset\} \cup \{A \subseteq X | A^c\ is\ finite\}$ then show that τ is topology on X.
2. Let, $\mathbb{N}$ be a set of natural numbers. Define $\tau = \{\emptyset, \mathbb{N}\} \cup \{A_n | n = 1,2,3, \ldots\}$, where $A_n = \{1,2, \ldots, n\}$. Then show that τ is topology on $\mathbb{N}$.
3. If X be an uncountable set and $\tau = \{\emptyset\} \cup \{A \subseteq X | A^c\ is\ countable\}$ then show that τ is topology on X.
4. For any set A in topological space (X, τ), show that $\bar{A} = A \cup d(A)$.
5. Prove that $(\mathbb{R}, \tau_u)$ is a normal space.
6. Prove that, a subset of usual topological space is compact if and only if it is closed and bounded.
7. Show that in any topological space (X, τ) following properties holds,

1) $\emptyset$ *and* X are closed sets

2) Finite union of closed sets is closed.

3) Arbitrary intersection of closed sets is closed.

8. Find derived set of (a, b) in usual topological space.

9. Find $d(\mathbb{N})$ in indiscrete topological space.

10. Prove that union of two compact topological space is compact but intersection of two compact spaces need not be compact.

11. Show that, intersection of two compact spaces need not be compact.

12. Prove that, usual topological space is not compact.

13. For any set A in topological space (X, τ), prove that $\bar{A} = A \cup d(A)$.

14. Let (X, τ_1) and (Y, τ_2) be two topological spaces and $f: X \to Y$ be continuous. Prove that, $f(c(A)) \subseteq C^*(f(A))$ where $A \subseteq X$ and $C \text{ and } C^*$ denotes closure in $X \text{ and } Y$ respectively.

15. Show that every closed bounded interval in usual topological space is compact.

16. If (X, τ_1) and (Y, τ_2) are two topological spaces. Then show that $f: X \to Y$ is continuous if and only if inverse image of every τ_2- closed set is τ_1- closed set.

17. If (X, τ_1) and (Y, τ_2) are two topological spaces. Then show that $f: X \to Y$ is continuous if and only if inverse image of every τ_2- open set is τ_1- open set.

18. Let (X, τ_1) and (Y, τ_2) be two topological spaces and $f: X \to Y$ be continuous. Prove that, 1) $c[f^{-1}(B)] \subseteq f^{-1}[c^*(B)]$ and

2) $f^{-1}[i^*(B)] \subseteq i[f^{-1}(B)]$.

19. Prove that, the property of space being first countable space is hereditary property.

20. Prove that, a topological space (X, τ) is compact if and only if any family of closed sets having finite intersection property has non-empty intersection.

21. Let, (X, τ) be a topological space and $A, B \subseteq X$. Prove that

1) $i(\emptyset) = \emptyset, i(X) = X, i\big(i(A)\big) = i(A)$;

2) if $A \subseteq B$ then $i(A) \subseteq i(B)$ and 3) $i(A \cap B) = i(A) \cap i(B)$.

22. Prove that, usual topological space is second countable space.

23. Prove that, every second countable space is first countable space.

24. Prove that, in second countable space every open covering of a subset is reducible to countable sub covering.

25. Show that, a space being second topological space is topological property.

26. Show that, a subset E of real line $\mathbb{R}$ is connected if and only if $E$is an interval.

27. Let X be a non-empty set and i^* is interior operator on X. Let $\tau = \{A \subseteq X: i^*(A) = A\}$ then show that τ is topology on X and $i^*(A) = i(A)$.

28. Give an example of non T_2 −space in which every convergent sequence has unique limit.

29. Show that any finite subset of topological space (X, τ) is compact.

30. Prove that, a topological space (X, τ) is T_0 −space if and only if given any distinct points x, y in X either $x \notin \overline{\{y\}}$ or $y \notin \overline{\{x\}}$.

Q. Short Answer Questions.

1. Let X be an infinite set. Define $\tau = \{A \subseteq X: A^c \textit{ is finite}\}$. Then show that τ is topology on X.
2. Let X be an countable set. Define $\tau = \{A \subseteq X: A^c \textit{ is countable}\}$. Then show that τ is topology on X.
3. Find $d(\mathbb{N})$ in usual topological space.
4. Let, $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}\}$ be topology on X. Then find $d(\{a\})$.
5. Show that, a set A is open in topological space (X, τ) if and only of $i(A) = A$.
6. In any topological space (X, τ), prove that $\bar{A}$ is the smallest closed set containing A.
7. In topological space (X, τ), show that a set is open if and only if it is neighbourhood of each of its points.
8. In any topological space (X, τ), prove that $i(A)$ is the largest open set contained in A.
9. Let $|X| \geq 2$, (X, τ) be a discrete topological space and A be any subset of X. Find $d(A)$.
10. Show that every metric space is regular space.
11. Show that being space is topological property.
12. Let F be a closed set in topological space (X, τ) and $x \notin F$ then prove that there exist an open set G in X such that $x \in G \subseteq F^c$.
13. Prove that, in T_2 −space every convergent sequence has unique limit
14. Show that, every continuous mapping of compact space into Hausdorff space is closed.

15. Show that every compact subset of T_2space is closed.
16. Give an example of every seperable space need not be Lindelof space.
17. Let (X, τ) be a topological space and Y be any non-empty subset of X. Define, $\tau^* = \{G \cap Y | G \in \tau\}$ then show that τ^* is a topology on Y.
18. Show that every second countable space is separable.
19. Prove that, continuous image of separable space is separable so property of being separable space is a topological property.
20. If (X, τ_1) and (Y, τ_2) are two topological spaces and $f: X \to Y$ is bijective, then show that f is homeomorphism if and only if $f(i(A)) = i^*(f(A))$; $\forall A \in X$.
21. If (X, τ_1) and (Y, τ_2) are two topological spaces. Then prove that, $f: X \to Y$ is open map if and only if $f(i(A)) \subseteq i^*(f(A))$; $\forall A \in X$.
22. Show that, every compact topological space has $Bolzano - Weierstrass$ property.
23. Show that being regular space is hereditary property.
24. Prove that, a topological space (X, τ) is T_1-space if and only if every singleton subset of X is closed.
25. Show that any metric space is normal.
26. Prove that, every second countable space is Lindelof space.
27. Give an example of to show that every Lindelof space need not be second countable space.
28. Prove that, a closed subspace of Lindelof space is Lindelof space.
29. Show that, space being Lindelof space is not hereditary property.
30. Show that the product space $(X \times Y, \tau)$ is second axiom space if and only if both (X, τ_1) and (Y, τ_2) are second axiom spaces.
31. Show that every T_4 −space is T_3 −space.
32. Prove that, the property of space being T_2 −space is preserved under bijective, open map.
33. Prove that, a topological space (X, τ) is regular space if and only if the family of closed neighbourhood of any point of X forms local base at that point.
34. Show that every compact subset of T_2 −space is closed.
35. Show that the regular space is need not be T_2 space.

36. Prove that, a topological space (X, τ) is regular space if and only if the family of closed neighborhood of any point of X forms local base at that point.

37. Show that, every closed continuous image of normal is normal.

38. Show that any metric space is normal.

39. Show that every regular, second axiom space is completely normal.

40. Show that every completely normal space is normal.

41. Show that the usual topological space is not compact space.

42. Show that T_3 space need not be a T_4 space.

43. Give an example to show that subspace of a normal space need not be normal.

44. Give an example of a normal space which is not regular.

45. Prove that, every completely regular space is regular.

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Department of Mathematics
M.Sc. I (Semester-II)
Complex Analysis-MMT 203
Subject Code-96108

1) Define following terms.

1) Harmonic function
2) Smooth path
3) Index of a closed curve
4) Entire function
5) Removable singularity
6) Fixed point
7) Function of bounded variation
8) Zeros of an analytic function
9) Pole of a function
10) Meromorphic function
11) Radius of convergence of the power series
12) Smooth path
13) Total variation
14) Singularities
15) Non-isolated singularity
16) Essential singularity
17) Normal family
18) Locally bounded family
19) Region
20) Piecewise smooth path
21) Analytic function
22) Mobius transformation
23) Trace of path
24) Rectifiable path
25) Isolated singularity

2) Fill in the blanks.

1) Period of e^z is…

2) A Mobius transformation has zero and infinity as its any fixed points if and only if it is …

3) The value of the integral $\int_\gamma \frac{1}{z-a} dz$ where $\gamma(t) = a + re^{it}, 0 \leq t \leq 2\pi$ is…

4) If $\gamma(t) = a + e^{10\pi it}$; $0 \le t \le 1$ then $\eta(\gamma; a) = \cdots$

5) A function f that is analytic except for finite number of points is called…

6) Fixed point of the transformation $S(z) = \frac{z-1}{2}$ is…

7) The cross ratio $(1, 0, i, 1) = \cdots$

8) Let $f(z) = \frac{\cos z}{z^2+1}$, then $f(z)$ has singularity at …

9) The value of the integral $\int_\gamma \frac{1}{z-a} dz$ where $\gamma(t) = a + re^{it}, 0 \le t \le 2\pi$ is…

10) A Set $F \subset C(G, \Omega)$ is normal if and only if …

11) Radius of convergence for the power series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ is…

12) The cross ratio $(0, 1, i, -1) = \cdots$

13) Let f is an entire function. If real part of $f(z)$ is bounded then the function f is…

14) If f has an isolated singularity at a then the point $z = a$ is a removable singularity if and only if $\lim_{z \to a} (z - a)f(z) = ...$

15) If $\gamma(t) = a + e^{in\pi t}$; $0 \le t \le 1$ then $\eta(\gamma; a) =$…

16) The value of cross ratio $(7 + i, 1, 0, \infty)$=…

17) If the cross ration (z_1, z_2, z_3, z_4) is real then all four points lies on…

18) A function $f(z) = e^{\frac{1}{z}}$ has… singularity at $z = 0$.

19) If f is nonconstant analytic function on G then f attains its maximum value at…

20) If γ is piecewise smooth and $f: [a, b] \to C$ is continuous then $\int_a^b f d\gamma = \cdots$

21) If G is a region and $f: G \to C$ is an analytic function such that there is a point a in G with $|f(a)| \ge |f(z)|$ for all $z \in G$. Then f is...

22) If f be an analytic on $B(a; R)$ and $|f(z)| \le M$ for all $z \in B(a; R)$. Then $|f^n(a)| \le \cdots$

23) $f(z) = \frac{1}{sinz - cosz}$ at $z = \frac{\pi}{4}$ has pole of order…

24) A function $f(z) = \sin\left(\frac{1}{z}\right)$ has…at z=0.

25) Let G be an open set and $f: G \to C$ an analytic function. If γ is a closed rectifiable curve in G such that $\eta(\gamma; w) = 0$ for all $w \in C - G$. Then for $a \in G - \{\gamma\}$ $f^{(k)}(a)\eta(\gamma; a) = \cdots$

3) Short answer questions.

1) Show that the function $f(z) = z^3 - 6z + 8$ has 3 zeros in $B(0;3)$.

2) Compute $\int_{|z|=2} \frac{z^5-1}{z^6-6z+5} dz$.

3) State and prove Hurwit'z theorem.

4) Show that the function $u(x,y) = x + e^{-x}\cos y$ is harmonic. Find harmonic conjugate and corresponding analytic function.

5) Find the Mobius transformation which maps $(-1,0,1)$ onto $(-i,0,i)$.

6) Show that if f is bounded entire function then it is constant.

7) If $f: G \to \mathbb{C}$ is an analytic function and γ is closed rectifiable curve such that $\eta(\gamma;\omega) = 0$ for all $\omega \in \mathbb{C} - G$ then show that $\int_\gamma f = 0$.

8) Let G be a region and f is a non constant analytic function on G then show that any open set U in G, $f(U)$ is open.

9) Let $f: D \to D$, be a one-one analytic map of $D = \{z: |z| < 1\}$ onto itself and suppose $f(a) = 0$ then show that there is a complex number c with $|c| = 1$ such that $f = c\emptyset_\alpha$.

10) Let f be meromorphic in G with pole $p_1, p_2, \dots, p_m$ and zeros $z_1, z_2, \dots, z_n$ counted according to the multiplicity. If γ is a closed rectifiable curve in G with $\gamma \cong 0$ and not passing through $p_1, p_2, \dots, p_m$ and zeros $z_1, z_2, \dots, z_n$ then show that $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)} dz = \sum_{k=1}^{n} n(\gamma, z_k) - \sum_{k=1}^{m} n(\gamma, p_k)$.

11) Show that Mobius transformation has infinity as its only fixed point if and only if it is a translation.

12) Show that the function $u(x,y) = sinxcoshy$ is harmonic. Find harmonic conjugate and corresponding analytic function.

13) Calculate residue of $\frac{z^2}{(z-1)(z-2)^2}$.

14) Evaluate $\int_{|z|=2} \frac{2z+1}{z^2+z+1} dz$.

15) Show that the function $f(z) = z^3 - 6z^2 + 3z + 1$ has 2 zeros in $B(0;3)$.

16) Let f be analytic in B(a; R) and $|f(z)| \leq M$ for all z in B(a; R) then prove that $|f^n(a)| \leq \frac{n!M}{R^n}$.

17) Evaluate $\int_\gamma \frac{e^{iz}}{z^2} dz, \gamma(t) = e^{it}, 0 \leq t \leq 2\pi$.

18)Show that if $f: G \to C$ is differentiable at a point a in G then f is continuous at a.

19) Show that Mobius transformation has infinity and zero are its fixed point if and only if it is a dilation.

20) Show that Mobius transformation other than identity map has at most two fixed points.

21) Find the Mobius transformation which maps $(-1, i, 1)$ onto $(-1, 1, 2+\ i)$.

22) Show that Mobius transformation takes circle onto circle.

23) Show that the function $u(x,y) = x^2 - y^2$ is harmonic. Find harmonic conjugate and corresponding analytic function.

24) Show that the function $u(x,y) = e^x(xsiny + y\cos y)$ is harmonic. Find harmonic conjugate and corresponding analytic function.

25) Let $\gamma: [a,b] \rightarrow G \subseteq C$ be a function. Show that γ is of bounded variation if and only if Re γ and Imγ are of bounded variation.

26) $\gamma: [0,2\pi] \rightarrow C$ defined by $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$ for all integers n then evaluate $\int_\gamma z^n dz$.

27) $\gamma: [0,2\pi] \rightarrow C$ defined by $\gamma(t) = e^{int}, 0 \leq t \leq 2\pi$ for all integers n then evaluate $\int_\gamma \frac{1}{z} dz$.

28) Let $f$be analytic in the disk $B(a,R)$ and suppose that γ is closed rectifiable curve in B(a, R) $\int_\gamma f = 0$.

29) Let f is an entire function if real part of $f(z)$ is bounded then show that The function f is constant.

30) Find the Mobius transformation which maps $(0, 1, \infty)$ onto $(-1, 0, 1)$.

31) Find the Mobius transformation which maps $(0, 1, \infty)$ onto $(1, \infty, 0)$.

32) Evaluate the cross ratio $(2, 1-i, 1, 1+i)$.

33) Evaluate the cross ratio $(i-1, \infty, 1+i, 0)$.

34) Show that Mobius transformation preserves the cross ratio.

35) An isolated singularity of function f at $z = a$ is removable singularity if and only if f is bounded in the neibourhood of a.

36) Show that an isolated singularity of function f is removable singularity if and only if $\lim_{z\to a} f(z) = c$.

37) Identify singularities and discuss its types of the function $f(z) = \frac{1}{z(z-1)^3(z-i)^5}$.

38) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for region $|z| < 1$.

39) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for region $|z| > 3$.

40) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for region $1 < |z| < 3$.

41) Calculate residue of $\frac{z+1}{z^2-2z}$.

42) Discuss the singularities of the function $\frac{sinz}{z}$

43) Show that a Mobius transformation is uniquely determined by its action on any three given points in C_∞.

44) Let T be a Mobius transformation with fixed points z_1 and z_2. If S be a Mobius transformation, show that $S^{-1}TS$ has fixed point $S^{-1}z_1$and $S^{-1}z_2$.

45) Show that cross ratio is invariant under the Mobius transformation.

4) Long answer questions.

1) For a given power series $\sum_{n=0}^{\infty} a_n(z-a)^n$ define the number R,

$0 \le R \le \infty$ by $\frac{1}{R} = \lim_{n\to\infty} sup|a_n|^{\frac{1}{n}}$. Show that

i) if $|z-a| < R$ then the series converges absolutely.

ii) if $|z-a| > R$ then the terms of the series become unbounded and so the series diverges.

iii) If $0 < r < R$ then the series converges uniformly on $A = \{z \mid |z| \le r\}$.

iv) The number R is the only number having properties (i) and (ii).

2) Let G be an open set in $\mathbb{C}$ and let γ be a rectifiable path in G with initial and end points α and β respectively. If $f: G \to \mathbb{C}$ is a continuous function with a primitive $F: G \to \mathbb{C}$ then show that $\int_\gamma f = F(\beta) - F(\alpha)$.

3) Prove that if G is a region and $f: G \to \mathbb{C}$ is an analytic function such that there is a point a in G with $|f(a)| \ge |f(z)|$ for all $z \in G$ then f is constant.

4) Show that a non constant polynomial has zero.

5) Prove that if G be a region and $f: G \to \mathbb{C}$ be continuous function such that $\int_T f = 0$ for every triangular path T in G then f is analytic function.

6) Evaluate $\int_0^\pi \frac{1}{a+\cos\theta} d\theta$ $(a > 1)$.

7) If G is open and connected and $f: G \to \mathbb{C}$ is differentiable with $f'(z) = 0$ for all z in G then show that f is constant.

8) If S is Mobius transformation then show that S is the composition of translation, dilations and the inversion.

9) If $\gamma: [0, 1] \to \mathbb{C}$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then show that $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z-a}$ is an integer.

10) Define entire function. State and prove Liouville's theorem.
11) Let G be an open set and $f: G \to \mathbb{C}$ be a differentiable function then show that f is an analytic on G.
12) If f has an essential singularity at $z = a$ then show that for every $\delta > 0$, $f(ann(a;\ 0, \delta))$ is dense in $\mathbb{C}$.
13) State and prove Morera theorem.
14) State and prove Goursat's theorem.
15) Show that polynomial with no zero is constant.
16) Prove that if G is a region and $f: G \to \mathbb{C}$ is an analytic function such that there is a point a in G with $|f(a)| \leq |f(z)|$ for all $z \in G$ then either $f(a) = 0$ or f is constant on G.
17) Let γ be closed rectifiable curve in C. Then show that $\eta(\gamma; a)$ is constant for a belonging to a component of $G = C - \{\gamma\}$. Also $\eta(\gamma; a) = 0$ for a belonging to the unbounded component of G.
18) Evaluate $\int_0^{2\pi} \frac{1}{1+a\sin\theta} d\theta \ (-1 < a < 1)$.
19) State and prove open mapping theorem.
20) Evaluate $\int_0^{\infty} \frac{x}{1+x^4} dx$.
21) State and prove Rouche's theorem.
22) State and prove Taylors theorem.
23) Evaluate $\int_0^{\infty} \frac{1}{1+x^2} dx$.
24) Let G be an open set and $f: G \to C$ an analytic function. If γ is a closed rectifiable curve in G such that $\eta(\gamma; w) = 0$ for all $w \in C - G$. Then show that for $a \in G - \{\gamma\}$ $f^{(k)}(a)\eta(\gamma; a) = k! \frac{1}{2\pi i} \int_\gamma \frac{f(z)}{(z-a)^{k+1}} dz$.
25) If γ is piecewise smooth and $f: [a, b] \to C$ is continuous then show that $\int_a^b f d\gamma = \int_a^b f(t)\gamma'(t)dt$.
26) State and prove Riemann mapping theorem.
27) Suppose f and g are meromorphic in neighbourhood of $\bar{B}(a;R)$ with no zeros or pole on the circle $\gamma = \{z : |z-a| = R\}$ if z_f, z_g (p_f, p_g) are the number of zeros (poles) of f and g inside γ counted according to their multiplicity and if $|f(z)+g(z)| < |f(z)| + |g(z)|$ on $\{\gamma\}$ then show that $z_f - z_g = p_f - p_g$.
28) Let f and g are analytic on G and Ω respectively with $f(G) \subseteq \Omega$ then show that $(g \circ f)' = g'(f(z))f'(z)$.
30) State and prove Montel's theorem.

MATHEMATICS
Differential Geometry (MMT-205)
Subject Code: 96110

Q.1	**a)**	**Define following terms**
	1)	Tangent Space.
	2)	Velocity vector of a curve.
	3)	Principal Normal Vector Field.
	4)	Coordinate Patch.
	5)	Shape operator of Surfaces.
	6)	Tangent Vector Field.
	7)	Regular curve.
	8)	Unit tangent vector field.
	9)	Surface.
	10)	Monge Patch.
	11)	Tangent vector in $\mathbb{R}^3$.
	12)	Curves in $\mathbb{R}^3$.
	13)	Binomial vector field.
	14)	Orthogonal Transformation.
	15)	Gauss map.
	16)	
	17)	
	18)	
	19)	
	20)	
	21)	
	22)	

	23)	
	24)	
	25)	
	b)	**Fill in the blanks**
	1)	If $\overline{w} = x^2U_1 + yz\,U_2$ and $\bar{v} = (-1,0,2)$ and p $(2,1,0)$ then $\nabla_{\bar{v}}\overline{w}$ at p is ……..
	2)	A mapping X:D→ $\mathbb{R}^3$ is a regular iff………………
	3)	If K(P) $>$ 0 then quadratic approxiamate is ……..
	4)	In a surface M, its Gaussian curvature is zero then M is called…..
	5)	Let f $=$ e^xcosy and $\bar{v} = (2,-1,3)$ and $p = (2,0,-1)$ then $\overline{v_p}$ [f] is…….
	6)	Let $\overline{V} = x\overline{U_1} + \overline{U_2}$ and $\overline{W} = x^2\overline{U_1}$ be two vectors fields, then the value of $\overline{W} - x\overline{V}$ at point p (-1,0,2) is
	7)	A vector field $\overline{Y}$ on a curve is parallel iff its derivative is …….
	8)	A mapping X:D→ $\mathbb{R}^3$ is a regular iff………………
	9)	Let f $=$ ${x_1}^2 + 2x_2x_3$, and $\bar{v} = (2,1,-3)$, $p = (0,-1,3)$ then $\overline{v_p}$ [f] is….
	10)	Let $\overline{T}$ is a tangent vector field and $\overline{N}$ is a principle normal vector field of unit speed curve β. Then Binomial vector field of β is given as….
	11)	$df = \sum_i \frac{\partial f}{\partial x_i} dx_i$ represents …….
	12)	For a straight line curvature k is ……
	13)	If $\overline{T}$ is a translation by a then $\overline{T}^{-1}$ is a translation by ……..
	14)	In a surface M, its Gaussian curvature is zero then M is called…..
	15)	Let $\beta(s) = (\frac{4}{5}\cos s, 1 - \sin s, \frac{-3}{5}\cos s)$ then curvature k is ……..
	16)	
	17)	
	18)	
	19)	
	20)	
	21)	

	22)	
	23)	
	24)	
	25)	
Q.2	**1)**	If $\bar{F}$ is an isometry of $\mathbb{R}^3$such that $\bar{F}(\bar{0}) = \bar{0}$ then show that $\bar{F}$ is an orthogonal transformation.
	2)	Find the Gaussian and mean curvature for the surface X (u, v) = (u cosv, u sinv, bv); b≠ 0.
	3)	If $\bar{V}$ & $\bar{W}$ are vector fields on $\mathbb{R}^3$ and f, g, h are real valued functions then show that i)(f$\bar{V} + g\bar{W}$)[h]=f $\bar{V}[h] + g\bar{W}$[h] ii) $\bar{V}[af + bg] = a\bar{V}[f] + b\bar{V}[g]$ iii) $\bar{V}[fg] = \bar{V}[f]g + f\bar{V}[g]$
	4)	If $\bar{v}$ & $\bar{w}$ are linearly independent unit tangent vector at point p to surface M then prove that i)S ($\bar{v}$) × S($\bar{w}$)=K(p) ($\bar{v}$ × $\bar{w}$) ii) S ($\bar{v}$) × $\bar{w} + \bar{v}$ × S ($\bar{w}$)= 2H(p) ($\bar{v}$ × $\bar{w}$)
	5)	Prove that regular curve α with k>0 is a cylindrical helix iff the ratio $\frac{\tau}{k}$ = constant.
	6)	For a patch X:D→ $\mathbb{R}^3$, if E=$X_u.X_u$,$F = X_u.X_v$ *and* $G = X_v.X_v$ then prove that X is a regular iff EG – F2 ≠ 0 .
	7)	Show that $\bar{u}.\bar{v} \times \bar{w} \neq 0$ iff $\bar{u},\bar{v},\bar{w}$ are linearly independent.
	8)	Let $\bar{V} = (1,2,-1)$, $\bar{W}$= (-1,0,3) be a tangent vector at point p then find $\frac{\bar{V}}{\|\|\bar{V}\|\|}$.
	9)	Find the unit speed of reparameterization of helix α defined by $\alpha(t)$= (acost, asint, bt); b≠ 0 .
	10)	Let β be the unit speed curve in $\mathbb{R}^3$ with curvature k>0 then show that vector field $\bar{T}$, $\bar{N}$, $\bar{B}$ forms a frame at every point on the curve β.
	11)	Find the Frenet apparatus for curve $\alpha(t) = (t - cost, sint, t)$ at t =0.
	12)	Prove that a non-constant curve is a straight line iff its acceleration is zero.
	13)	Find the principal curvatures for a right circular cylinder.
	14)	If $\bar{C}$:$R^3 \to R^3$ is orthogonal transformations then show that $\bar{C}$ is an isometry.
	15)	If $\emptyset = f_1dx_1 + f_2dx_2 + f_3dx_3$ is 1-form, then find its exterior derivative

		$d\emptyset$.Also show that $d^2\emptyset$=0.
	16)	Show that the curves given by $\alpha(t) = (t, 1 + t^2, t), \beta(t)\ (sint, cost, t), \gamma(t)(sinht, cosht, t)$ have the same initial velocity $\bar{v}_p$. Also compute $\bar{v}_p[f]$ by evaluating $f = x^2 - y^2 + z^2$ on each of the curves.
	17)	Let $\bar{e}_1, \bar{e}_2, \bar{e}_3$ be a frame at appoint p of $\mathbb{R}^3$. If $\bar{v}$ is any tangent vector to $\mathbb{R}^3$ at p then show that $\bar{v} = (\bar{v}.\bar{e}_1)\bar{e}_1 + (\bar{v}\,.\bar{e}_2)\bar{e}_2 + (\bar{v}\,.\bar{e}_3)\bar{e}_3$.
	18)	Show that a rectangular strip bent into figure 8 is not a surface.
	19)	Obtain Frenet approximation β of near s=0 for the unit speed curve.
	20)	If $\bar{W} = x\bar{U}_1 + x^2\bar{U}_2 - z^2\bar{U}_3$ then find the covariant derivative of $\bar{W}$ with respect to $\bar{v}$ at point p= (1,3, -1) where $\bar{v}$ = (1, -1, 2).
	21)	Show that rotation $\bar{c}$ around z axis is an isometry.
	22)	If $\bar{T}$ is translation by a then show that $\bar{T}$ has an inverse $\bar{T}^{-1}$ which is translation by -a.
	23)	Show that the sphere in $\mathbb{R}^3$ is a surface.
	24)	Prove that a curve α in a surface M is a straight line if and only if α is a both geodesic and asymptotic.
	25)	For which values of c is M: $z(z-2) + xy = c$ is a surface?
	26)	Show that the shape operator describes the cylindrical surface as half flat and half round.
	27)	Find the Gauss map for the saddle surface M: z=xy.
	28)	Show that every point of a surface M:$x^2 + y^2 + z^2 = r^2$ is umbilic.
	29)	If $\emptyset$ is a 1-form on $\mathbb{R}^3$, then prove that $\emptyset = \sum_i f_i\, dx_i$, where $f_i = \emptyset(U_i)$.
	30)	If $\bar{X}, \bar{Y}$ and $\bar{V}$ are vector fields in $\mathbb{R}^3$, then prove that $\bar{V}[\bar{X}.\bar{Y}]$=$\sum_i x_i\, \bar{V}[y_i] + \sum_i \bar{V}[\,x_i]\, y_i$.
	31)	Let $\bar{v}$ and $\bar{w}$ be tangent vectors at the same point p. Prove that $\bar{v} \times \bar{w}$ is orthogonal to $\bar{v}$ and $\bar{w}$ and has length $\|\,\bar{v} \times \bar{w}\|^2$= $(\bar{v}.\bar{v})(\,\bar{w}.\bar{w})$-$(\bar{v}.\bar{w})^2$

	32)	Prove that $\bar{v} \times \bar{w} \neq 0$ if and only if $\bar{v}$ and $\bar{w}$ are linearly independent and also show that $\|\bar{v} \times \bar{w}\|$ is the area of the parallelogram with sides $\bar{v}$ and $\bar{w}$.
	33)	Let β be a unit speed curve in $\mathbb{R}^3$ with $\kappa > 0$ then show that β is plane curve if and only if its torsion is zero.
	34)	If $\sum$ is a sphere of radius 'a' with centre at origin of $\mathbb{R}^3$ then show that every spherical unit speed curve β has a curvature $\kappa \geq \frac{1}{a}$
	35)	If $\bar{W} = x^2\bar{U}_1 + yz\bar{U}_3$ and $\bar{v} = (-1,0,2)$, $\bar{p} = (2,1,0)$ then find $\nabla_{\bar{v}}\bar{W}$ at p.
	36)	Prove that translation is an isometry.
	37)	If $\bar{S}$ and $\bar{T}$ are translations then show that $\bar{S}\bar{T} = \bar{T}\bar{S}$ is also a translation.
	38)	Show that the cylinder in $\mathbb{R}^3$ is a surface.
	39)	Prove that $M: (x^2 + y^2)^2 + 3z^2 = 1$ is a surface.
	40)	Find a non vanishing normal vector field on $M: z = xy$ and also find two linearly independent tangent vector fields at each point of M.
	41)	Show that the shape operator of a plane surface is zero.
	42)	Find the principal curvatures for a right circular cylinder.
	43)	Find the quadratic approximation near origin for the surface $M: z = e^{x^2+y^2} - 1$
	44)	
	45)	
	46)	
	47)	
	48)	
	49)	
	50)	
