Rayat Shikshan Sanstha's

YASHAVANTRAO CHAVAN INSTITUTE OF SCIENCE, SATARA

(An Autonomous College)

Reaccredited by NAAC with 'A+' Grade

Bachelor of Science

Part - II

Mathematics

Syllabus

to be implemented w.e.f. June, 2022

Structure of the Course:

2) Semester III

Sr. No.	Subject title	Theory					Practical	
		Course No. and Course code	Title of Course	No. of lectures per week	Credits		No. of lectures per week	Credits
1.	Mathematics	Course V BMT 301	Real Analysis I	6	4	Practical III BMP 303	8	4
		Course VI BMT 302	Algebra I	6				

2) Semester IV

Sr. No	Subject title	Theory					Practical	
		Course No.and Course Code	Title of Course	No. of lectures per week	Credits		No. of lectures per week	Credits
1.	Mathematics	Course VII BMT 401	Real Analysis II	6	4	Practical IV BMP 403	8	4
		Course VIII BMT 404	Algebra II	6				

B.Sc. II: Evaluation structure

Semester III.

	ESE	Internal Exam		Practical			Submission	Total
		ISE-I	ISE-II		Exam	Journal	Seminar + Student Performance	
Course V	30	5	5	Practical-III(A)	25	5	5	150
Course VI	30	5	5	Practical IV(A)	25	5	5	

Semester IV

	ESE	Internal Exam		Practical			Submission	Total
		ISE-I	ISE-II		Exam	Journal	Industrial visit/Educational Tour + Student Performance	
Course V	30	5	5	Practical-III(A)	25	5	5	150
Course VI	30	5	5	Practical IV(A)	25	5	5	

Structure and Titles of the Course of B.Sc. II Course

Semester III

Code	Name of Course	Units
BMT 301	**Real Analysis I** (CREDITS:02; TOTAL HOURS: 45)	**Unit I:** Set and Functions **Unit II:** Completeness Property of $\mathbb{R}$ **Unit III:** Sequence of Real Numbers **Unit IV:** Monotone Sequences and Cauchy Sequences
BMT 302	**Algebra I** (CREDITS:02; TOTAL HOURS : 45)	**Unit I:** Matrices **Unit II**: Divisibility in Integers **Unit III:** Relation **Unit IV:** Groups

Semester IV

BMT 401	**Real Analysis II** (CREDITS:02; TOTAL HOURS: 45)	**Unit I:** Limit Superior and Inferior of Sequences **Unit II**: Series of Real Numbers **Unit III:** Sequence and Series of functions **Unit IV:** Differentiability and Integrability of Series of functions
BMT 402	**Algebra II** (CREDITS:02; TOTAL HOURS: 45)	**Unit I:** Subgroups **Unit II:** Cyclic Groups **Unit III:** Normal Groups **Unit IV:** Homomorphism and Permutation Group

Course V – BMT 301 **Real Analysis I**

Course Objectives: Student will be able to…

1. Understand basic statements and able to write basic proofs according to principles of quantificational logic.
2. Understand thoroughly and precisely the concept of "limit" in its various forms.
3. Define functions between sets equivalent sets, finite, countable and uncountable sets.
4. Show whether sequence is converges or diverges.

Credits (Total Credits 2)	**SEMESTER-III** **BMT 301** **Real Analysis I**	**No. of hours per unit/credits**
UNIT - I	**Sets and Functions**	**(14)**
	1.1 Sets 1.1.1 Operations on sets, Cartesian product of sets, Relation. **1.2 Functions** 1.2.1 **Definitions:** Function, Domain, Co-domain, Range, Graph of a function, Direct image and Inverse image of a subset under a function. 1.2.2 **Theorem:** If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$ then $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$. 1.2.3 **Theorem:** If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$ then $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$. 1.2.4 **Theorem:** If $f: A \to B$and $X \subseteq A , Y \subseteq A$ then $f(X \cup Y) = f(X) \cup f(Y)$. 1.2.5 **Theorem:** If $f: A \to B$ and $X \subseteq A , Y \subseteq A$ then $f(X \cap Y) \subseteq f(X) \cap f(Y)$. 1.2.6 **Definitions:** Injective, Surjective and Bijective function (1-1 correspondence), Inverse function. 1.2.7 **Theorem:** Composition of two bijective functions is a bijective function. **1.3 Countable Sets** 1.3.1 **Definitions:** Finite sets, Infinite sets, Countable Sets, Uncountable Sets. 1.3.2 **Examples of Countable sets:** Set of Natural numbers, Set of Integers, Cartesian product of Countable sets. 1.3.3 **Theorem:** Countable union of countable set is countable. 1.3.4 **Theorem:** Set of Rational numbers is countable.	

	1.3.5 **Theorem:** Any subset of countable set is countable. 1.3.6 **Theorem:** The closed interval [0,1] is uncountable. 1.3.7 **Theorem:** The set of all real numbers is uncountable. 1.3.8 **Theorem:** Every infinite set has a countably infinite subset. 1.3.9 **Examples.**	
UNIT - II	**Completeness property of $\mathbb{R}$**	**(08)**
	2.1 Definitions: Lower bound, Upper bound of a subset of $\mathbb{R}$, Bounded set, Supremum $(l.u.b)$, Infimum $(g.l.b)$. **2.2 Least Upper Bound Axiom [Completeness Property of $\mathbb{R}$]:** **2.3 Theorem (Archimedean Property):** If $x \in \mathbb{R}$ then there exists $n_x \in \mathbb{N}$ such that $x \leq n_x$. 2.3.1 **Corollary:** If $S = \{\frac{1}{n} : n \in \mathbb{N}\}$ then inf $S = 0$. 2.3.2 **Corollary:** If $t > 0$ then there exist $n_t \in \mathbb{N}$ such that $0 < \frac{1}{n_t} < t$. 2.3.3 **Corollary:** If $y > 0$ then there exist $n_y \in \mathbb{N}$ such that $n_y - 1 < y < n_y$. **2.4 Theorem:** There exists a positive real number x such that $x^2 = 2$. 2.4.1 **Corollary:** If x and y are real numbers with $x < y$ then there exist an irrational number z such that $x < z < y$. **2.5 Intervals** 2.5.1 **Characterization Theorem:** If S is a subset of $\mathbb{R}$ that contains at least two points and has the property. If $x, y \in S$ and $x < y$ then the closed interval $[x, y] \subseteq S$ then S is an interval.	
UNIT - III	**Sequence of Real Numbers**	**(14)**
	3.1 Sequence and Subsequence: 3.1.1 Definition of Sequence and Subsequence. 3.1.2 Sequence of functions **3.2 Limit of a Sequence** 3.2.1 **Definition** 3.2.2 **Theorem:** If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers and if $\lim_{n\to\infty} S_n = L$ then $L \geq 0$. **3.3 Convergent Sequence** 3.3.1 **Theorem:** Convergent sequence cannot converge to two	

	distinct points. 3.3.2 **Theorem (Without Proof):** If sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L then any subsequence of $\{S_n\}_{n=1}^{\infty}$is also convergent to L. **3.4 Operations on Convergent sequences:** 3.4.1 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n + t_n) = L + M$. 3.4.2 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n - t_n) = L - M$. 3.4.3 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, f $c \in \mathbb{R}$ and if $\lim_{n\to\infty} s_n = L$ then $\lim_{n\to\infty} cs_n = cL$. 3.4.4 **Theorem:** If $0 < x < 1$ then the sequence $\{x^n\}$ converges to 0. 3.4.5 **Lemma:** If sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L then $\{{s_n}^2\}_{n=1}^{\infty}$ converges to L^2. 3.4.6 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n . t_n) = LM$. 3.4.7 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n / t_n) = L/M$.	
UNIT - IV	**Monotone Sequences and Cauchy Sequences**	**(09)**
	4.1 Monotone Sequence 4.1.1 Definition and Examples. 4.1.2 **Theorem:** A non-decreasing sequence which is bounded above is convergent. 4.1.3 **Theorem:** A non-increasing sequence which is bounded below is convergent. 4.1.4 **Corollary:** The sequence $\{(1 + \frac{1}{n})^n\}$ is convergent. 4.1.5 **Theorem (Without Proof):** A non-decreasing sequence which is not bounded above diverges to infinity. 4.1.6 **Theorem (Without Proof):** A non-increasing sequence which is not bounded below diverges to infinity. 4.1.7 **Theorem:** A bounded sequence of real numbers has convergent subsequence.	

	4.2 Cauchy Sequence 4.2.1 **Definition and Examples.** 4.2.2 **Theorem:** If sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges then $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence. 4.2.3 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$is bounded. 4.2.4 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$is convergent.	

Course outcomes: Student should be able to…

1. Recall properties of set, relations and functions.
2. Describe convergence of monotone sequences and completeness property of $\mathbb{R}$.
3. Apply convergence tests of sequences to identify the nature of sequence.
4. Derive the properties of Cauchy sequences and analyze their convergence.

References-

1. S.C. Malik and Savita Arora, Mathematical Analysis (Fifth Edition), New Age International (P) Limited, 2017(UNIT I, II, III, IV).
2. T. M. Apostol, Calculus (Vol.I), John Wiley and sons (Asia) P.Ltd.2002 (UNIT I, II, III, IV).
3. R.R. Goldberg, Methods of real Analysis, Oxford & IBH Publishing co. Pvt. Ltd, New Delhi 2019(UNIT I, II, III, IV).
4. R.G. Bartle and D.R. Sherbert, Introduction to Real Analysis, Wiley India Pvt. Ltd, Fourth Edition, 2016 (UNIT I, II, III, IV).
5. D.Somasundaram and B Choudhary, First Course in Mathematical Analysis, Narosa publishing house New, Delhi, Eighth Reprint 2013 (UNIT I, II, III, IV).
6. P.K. Jain and S.K. Kaushik, An Introduction to Real Analysis, S. Chand & Company Ltd. New Delhi, First Edition 2000 (UNIT I, II, III, IV).
7. Shanti Narayan and M.D. Raisinghania, Elements of Real Analysis, S. Chand & Company Ltd. New Delhi, Fifteenth Revised Edition 2014 (UNIT I, II, III, IV).

Course – VI BMT 302 **Algebra I**

Course Objectives: Student will be able to…

1. Understand types of Matrices and their applications
2. Develop the skills find the Eigen values and Eigen vectors
3. Present the divisibility and relationship between the Greatest common divisor and least common multiple
4. Present the concept Group and its basic properties

Credits (Total Credits 2)	SEMESTER-III BMT 302 Algebra I	No. of hours per unit/credits
UNIT - I	**Matrices**	(12)
	1.1 Introduction 1.1.1 Definition with Illustration. 1.1.2 Types of Matrices 1.1.3 Definitions: Transpose of Matrix, conjugate of Matrix, Symmetric Matrix, Asymmetric Matrix **1.2 Hermitian and Skew Hermitian** 1.2.1 **Definitions:** Hermitian and Skew Hermitian 1.2.3 **Theorem:** The necessary and sufficient condition for a matrix A to be Hermitian is that$A = A^{\theta}$. 1.2.4 **Theorem:** The necessary and sufficient condition for a matrix A to be Skew Hermitian is that $A^{\theta} = -A$. 1.2.5 **Theorem:** If A and B are Hermitian (Skew Hermitian) then A+B is also Hermitian (Skew Hermitian). 1.2.6 **Theorem:** If A is Hermitian then iA is Skew Hermitian. 1.2.7 **Theorem:** If A is Skew Hermitian then iA is Hermition. 1.2.8 **Theorem:** Every square Matrix is uniquely expressed as the sum of Hermitian Skew Hermitian matrix. **1.3 Eigenvalues and Eigenvectors** 1.3.1 **Definitions:** Minor of matrix, Rank of a matrix, Inverse of a matrix, Characteristics Polynomial of matrix 1.3.2 **Eigenvalues and Eigenvectors** 1.3.3 Examples on 1.3.2. **1.4 System of a linear Equations** 1.4.1 System of Homogeneous linear Equations. 1.4.2 Nature of solutions of $AX = 0$. 1.4.3 Examples on 1.4.1.	

	1.4.4 System of Non Homogeneous linear Equations. 1.4.5 Nature of Solution. 1.4.6 Examples on 1.4.4. **1.5 Cayley-Hamilton Theorem (Statement only)** 1.5.1 Applications of Cayley-Hamilton Theorem.	
UNIT - II	**Divisibility in Integers**	**(10)**
	2.1 Definition: Divisibility in integers **2.2 The well ordering principle (Statement Only)** **2.3 Properties of Divisibility** 2.3.1 Definition of divisor and Multiple 2.3.2 **Theorem:** Let a, b, c, d be integers. Then i) If a | b then a | bx ii) If a | b and a | c then a | bx+cy $\forall$ x, y $\in$ I iii) If a | b and b | c then a | c iv) If $m \neq 0$ is in $\mathbb{Z}$ and a | b $\Rightarrow$ am | bm v) If a | b and c | d then ac | bd vi) If ab | bc then a | c; $(b \neq 0)$ **2.4 Theorem: Division Algorithm (Without Proof)** **2.5 Greatest common divisor and least common multiple** 2.5.1 **Definition:** Greatest common divisor and least common multiple 2.5.2 **Theorem:** Let a and b be two integers at least one of them not 0. Then there exist a unique greatest common divisor d of a and b. Moreover, d can be written as $d = am + bn$ for integers m and n. 2.5.3 **The Euclidean Algorithm and Examples.** 2.5.4 **Definition**: Relatively Prime 2.5.5 **Euclid's lemma:** For a prime number p , if $p \mid ab$ then either $p \mid a$ or $p \mid b$. **2.6 Theorem: (Unique Factorization Theorem or Fundamentals Theorem of Arithmetic)**	
UNIT - III	**Relation**	**(12)**
	3.1 Relation 3.1.1 **Definitions:** Cartesian Product, Relation, Binary Relation, Inverse Relation. 3.1.2 Examples on 3.1.1. **3.2 Pictorial Representation of Relation** 3.2.1 Co-ordinate Diagram 3.2.2 Arrow Diagram 3.2.3 Matrix Representation 3.2.4 Directed Graph 3.2.5 Examples on 3.2.1 to 3.2.4 **3.3 Composition of Relations** 3.3.1 **Definition:** Composition of Relations	

	3.3.2 **Theorem:** Let A, B, C and D be sets. Let R: A⟶ B, S: B⟶ C, T: C⟶ D be Relation then $R \circ (S \circ T) = (R \circ S) \circ T$. **3.4 Types of Relations** 3.4.1 **Definitions:** Reflexive, Symmetric, Antisymmetric and transitive. 3.4.2 Examples on 3.4.1 3.4.3 **Theorem:** Let R be relation on set A. Then R^{∞} is the smallest transitive relation on A that contain R. **3.5 Equivalence relation and partitions** 3.5.1 **Theorem:** Let R be an equivalence relation on setA. Then quotient set A/R forms a partition of A. 3.5.2 **Theorem:** Let $\{A_i\}, i \in I$ be partition of a setA. Then there exists an equivalence Relation Ron the set A such that quotient set A / R is the given partition $\{A_i\}, i \in I$ on A. **3.6 Partial order relation**. 3.6.1 **Definition**: Partial order relation. 3.6.2 Examples on 3.6.1 **3.7 Congruence relation on Integers** 3.7.1 **Definition**: Congruence relation 3.7.2 **Congruence arithmetic** 3.7.3 **Theorem:** Let $n > 1$ be a fixed positive integer and a, b, c, d be arbitrary integers then the following conditions holds $i)$If $a \equiv b (mod n)$ and $c \equiv d (mod n)$then $a + c \equiv b + d (mod n)$. $ii)$If $a \equiv b (mod n)$ and $c \equiv d (mod n)$then $ac \equiv bd (mod n)$. 3.7.4 **Examples**	
UNIT - IV	**Groups**	**(11)**
	4.1 Binary operation on a set 4.1.1 **Definition:** Binary operation on a set with illustration **4.2 Semigroup** 4.2.1 **Definition:** Semigroup with illustration **4.3 Monoid** 4.3.1 **Definition:** Monoid with illustration **4.4 Group** 4.4.1 **Definition:** Group, Abelian Group, Finite Group,	

	Infinite Group, Order of a group 4.4.2 Examples on 4.4.1 **4.5 Properties of Groups** 4.5.1 **Theorem:** If $< G,*>$is a group, then a) Identity element in G is unique b) Every a in G has unique inverse in G. c)For every a in G, $(a^{-1})^{-1} = a$ d) For all a, b ∈ G, $(a * b)^{-1} = b^{-1} * a^{-1}$ 4.5.2 **Theorem:** If a, b, c are elements in a group G, then i) $a * b = a * c$ implies $b = c$ (Left Cancellation Law) ii)$b * a = c * a$ implies $b = c$ (Right Cancellation Law) 4.5.3 **Theorem:** If G is a group and a, b∈ G, then the equations $a * x = b$ and $y * a = b$have unique solutions $x = a^{-1} * b$ and $y = b^{-1} * a$respectively. 4.5.4 **Definition:** Order of element with illustration, Properties (Without Proof) **4.6 Permutations** 4.6.1 Definition with Illustration 4.6.2 Cyclic Permutation 4.6.3 Transposition, Disjoint Permutations, Even and Odd Permutations	

Course outcomes-Students should be able to…

1. Define matrices and its types, eigen values and eigen vectors and state Cayley Hamilton theorem.
2. Explain division and Euclidean algorithm with numerous examples.
3. Apply partial order relation and congruence relations to solve examples.
4. Classify groups into abelian, non abelian, finite and infinite groups and illustrate permutation groups with its applications

References:

1. Shantinarayan, A Text Book of Matrices, S. Chand Co. Pvt. Ltd. Raminagar, New Delhi,2010 (Unit I).
2. David. M. Burton, Elementary Number Theory ,7th Edition.2017 McGraw Hill Education (Unit II).
3. Schaum's Outline, Discrete Mathematics, (3rd Edition): Seymour hipschutz, Marehipson, Tata MaGraw-Hill Publishing Company Ltd., New Delhi.2010 (Unit III).
4. V.K. Khanna and S. K. Bhambri, A course in abstract Algebra, Vikas Publishing house Private Limited, New Delhi, Fifth Edition 2016 (Unit IV).

5. J.B. Fraleigh, A first course in abstract Algebra, Narosa Publishing House New Delhi, Tenth Reprint 2003.
6. A.R. Vasishtha, Modern Algebra, Krishna Prakashan, Meerut 1994.
7. M.Artin, Algebra, Prentice Hall of India, New delhi,1994.
8. I.N. Herstein, Topics in Algebra, Wiley India Pvt. Ltd,1975.

BMP 303 **Mathematics Practical III**

Course Objectives: Student will be able to…

1. Understand how matrices and determinants are used as mathematical tools in quality assurance.
2. Find power of Matrix and determining the inverse of matrix.
3. Use Matrices to represent a system of equations.
4. Provide exposure to problem solving through programming.

Credits (Total Credit 02)	**SEMESTER-III** **BMP 303** **Mathematics Practical III**	**No. of hours per unit/credits**
	Group A	
1	Solution of System of m linear homogeneous equations in n unknowns	
2	Solution of System of m linear non homogeneous equations in n unknown	
3	Inverse of Matrix by Cayley Hamilton Method	
4	Euclidean Algorithm	
5	Pictorial Representation of Relation	
6	Examples on equivalence relation	
7	Examples on Fermat's theorem	
8	Examples on Group & Order of an element	
9	Beta function	
10	Gamma function	
	Group B	
1	C-Introduction-I	
2	C-Introduction-II	
3	Complete Structure of C-programe	
4	Simple C-programmes	
5	If Statement, If else Statement & Switch Statement	
6	While loop & do while loop	
7	For loop	
8	Go to, break continue statement	
9	One Dimensional Array	
10	Two-Dimensional Array	

Course outcomes-Students should be able to…

1. Calculate solution to homogeneous and nonhomogeneous system of equations and Euclidean algorithm.
2. Simplify equivalence relations and problems on Gamma and Beta function.
3. Evaluate simple arithmetic operations using simple c programs.
4. Design computer programs using loops and arrays in C language.

Practical References-

1. Shantinarayan, A Text Book of Matrices, S. Chand Co., Pvt. Ltd. Raminagar, New Delhi Cambridge university press,2010.
2. V.K. Khanna and S. K. Bhambri, A course in abstract Algebra, Vikas Publishing house Private Limited, New Delhi, Fifth Edition 2016
3. R.B. Kulkarni, U.H. Naik, J.D. Yadav, S.P. Throat, A.A. Basade, H.V. Patil, H.T. Dinde, A Hand Book of Computational Mathematics Laboratory, Shivaji University Mathematics Society,2005
4. Shanti Narayan, P.K. Mittal: Integral Calculus, S. Chand and comp. New Delhi,2005.
5. B.P. Demidovich & I. A. Maron Computational Mathematics, translated by George Yankosky Mir Publishers, Moscow,1981.
6. J.B. Fraleigh, A first course in abstract Algebra, Narosa Publishing House New Delhi, Tenth Reprint, 2003.
7. A.R. Vasishtha, Modern Algebra, Krishna Prakashan, Meerut 1994
8. Seymour hipschutz, Marehipson Schaum's Outline, Discrete Mathematics, (3rd Edition), Tata MaGraw-Hill Publishing Company Ltd., New Delhi,2010.

SEMESTER- IV

Course VII- BMT401 **Real Analysis II**

Course Objectives: Student will be able to…

1. Understand the basic ideas required for their subsequent course work.
2. Study Limit Superior and Inferior of Sequences and tests for convergence of series.
3. Learn Sequence and series of functions which are useful in obtaining approximations to a given function and defining new functions from known ones.
4. Understand sequences whose terms are functions rather than real numbers and pay attention to the general properties that are associated with the uniform convergence of sequence and series of functions

Credits (Total Credits 2)	**SEMESTER-IV** **BMT401** **Real Analysis II**	**No. of hours per unit/credits**
UNIT - I	**Limit Superior and Inferior of Sequences**	**(14)**
	1.1 Definitions and Examples. **1.1.1 Theorem:** If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then $\lim_{n\to\infty} sup\, s_n = \lim_{n\to\infty} S_n$. **1.1.2 Theorem:** If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then $\lim_{n\to\infty} inf\, S_n = \lim_{n\to\infty} S_n$. **1.1.3 Theorem:** If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers and if $\lim_{n\to\infty} sup\, S_n = \lim_{n\to\infty} inf\, S_n = L$ where $L \in \mathbb{R}$ then $\{S_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n = L$. **1.1.4 Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers and if $s_n \le t_n$ then $i) \lim_{n\to\infty} sup\, s_n \le \lim_{n\to\infty} sup\, t_n$ $ii) \lim_{n\to\infty} inf\, s_n \le \lim_{n\to\infty} inf\, t_n$ **1.1.5 Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers then $i) \lim_{n\to\infty} sup\, (s_n + t_n) \le \lim_{n\to\infty} sup\, s_n + \lim_{n\to\infty} sup\, t_n$ $ii) \lim_{n\to\infty} inf\, (s_n + t_n) \ge \lim_{n\to\infty} inf\, s_n + \lim_{n\to\infty} inf\, t_n$.	

UNIT - II	Series of Real Numbers	(09)
	2.1 Convergent and Divergent Series. 2.1.1 **Definitions:** Convergent Series, Divergent Series and Examples. 2.1.2 If $\sum_{n=1}^{\infty} a_n$ is convergent series then $\lim_{n\to\infty} a_n = 0$. **2.2 Cauchy's General Principal for convergence (Statement only):** A necessary and sufficient condition for the convergence of an infinite series $\sum_{n=1}^{\infty} u_n$ is that the sequence of its partial sum $\{s_n\}$ is convergent. **2.3 Series of Nonnegative real numbers.** 2.3.1 **Definition and Examples.** 2.3.2 **Theorem:** A positive term series converges if and only if its sequence of partial sum is bounded above. **Theorem:** A positive term series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if $p > 1$. **2.4 Tests for convergence** 2.4.1 **Comparison Test (First Type)** If $\sum u_n$ and $\sum v_n$ are two positive term series and $k \neq 0$, a fixed positive real number (independent of n) and there exists a positive integer m such that $u_n \leq kv_n$ for every $n \geq m$ then $a)$ $\sum u_n$ is convergent if $\sum v_n$ is convergent and $b)$ $\sum v_n$ is divergent if $\sum u_n$ is divergent. 2.4.2 **Comparison Test (Second Type)** If $\sum u_n$ and $\sum v_n$ are two positive term series and there exist positive number m, such that $(u_n/u_{n+1}) \geq (v_n/v_{n+1})$ for every $n \geq m$ then $a)$ $\sum u_n$ is convergent if $\sum v_n$ is convergent and $b)$ $\sum v_n$ is divergent if $\sum u_n$ is divergent. 2.4.3 $\boldsymbol{p}$ **−Series Test:** The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$ 2.4.4 **Root Test:** Consider the series $\sum_{n=1}^{\infty} a_n$ .Then $a)$ If $\lim sup\|a_n\|^{1/n} < 1$ then the series convergent absolutely. $b)$ If $\lim sup\|a_n\|^{1/n} > 1$ then the series diverges. $c)$ If $\lim sup\|a_n\|^{1/n} = 1$, this test gives no information. **2.5 Alternating Series** 2.5.1 **Leibnitz Test:** If the alternating series $u_1 - u_2 + u_3 - u_4 + \ ...$, ($u_n > 0$ for every n) is such that $i)$ $u_{n+1} \leq u_n$, for every n and $ii)$ $\lim u_n = 0$ then the series converges. **2.6 Examples** **2.7 Absolute and Conditional Convergence** 2.7.1 Definition and Examples. 2.7.2 Theorem: Every Absolutely convergent series is	

	convergent. **2.8 Examples.**	
UNIT - III	**Sequence and Series of Functions**	**(14)**
	3.1 Pointwise convergence of sequence of functions: 3.1.1 Definition and Examples. **3.2 Uniform convergence of sequence of functions.** 3.2.1 Definition and Examples. **3.3 Uniform Convergence and Continuity:** 3.3.1 **Theorem:** Assume $f_n \to f$ uniformly on an interval S.If each function f_n is continuous at a point p in S then the limit function f is also continuous at p. 3.3.2 **Theorem:** If series of functions $\sum u_k$ converges uniformly to a function f on a set S and if each term u_k is continuous at a point p in S then f is also continuous at p.	
UNIT - IV	**Differentiability and Integrability of Series of functions**	**(08)**
	4.1 Theorem: Assume that$f_n \to f$ is uniformly on $[a, b]$ and assume that each function f_n is continuous on $[a, b]$. Define a new sequence $\{g_i\}$ by the equation $g_n(x) = \int_a^x f_n(t)\ dt$ if $x \in [a, b]$ and $g(x) = \int_a^x f(t)\, dt$. Then $g_n \to n$ uniformly on $[a, b]$ .In symbols, we have $\lim_{n\to\infty} \int_a^x f_n(t)\, dt = \int_a^x \lim_{n\to\infty} f_n(t)\, dt.$ **4.2 Theorem:** Assume that series of functions $\sum u_k$ converges uniformly to a function f on an interval $[a, b]$ where each u_k is continuous on $[a, b]$. For $x \in [a, b]$ define $g_n(x) = \sum_{k=1}^{n} \int_a^x u_k(t)\ dt$ and $g(x) = \int_a^x f(t)\ dt$. Then $g_n \to g$ uniformly on $[a, b]$. **4.3 Sufficient Condition for Uniform Convergence:** 4.3.1 **Theorem (Weierstrass M-Test):** Given series of functions $\sum u_k$ which converges pointwise to a function $f$on a set S.If there is a convergent series of positive constants $\sum M_n$ such that $0 \le \lvert u_n(x) \rvert \le M_n$; for every $n \ge 1$ and every x in S. Then $\sum u_k$ converges uniformly on S. **4.4 Power Series.** 4.4.1 **Definition.** 4.4.2 **Interval of Convergence and its examples**	

Course Outcomes: Student should be able to…

1. Calculate limit superior and inferior of sequences of real numbers.
2. Illustrate convergence of series of real numbers with the help of various tests of convergence.
3. Determine the convergence of sequence and series of functions in respect to pointwise and uniform convergence.
4. Derive the convergence of power series and build the interval of convergence.

References:

1. S.C. Malik and Savita Arora, Mathematical Analysis (Fifth Edition), New Age International (P) Limited, 2017(UNIT I, II, III, IV)
2. T. M. Apostol, Calculus (Vol.I), John Wiley and sons (Asia) P.Ltd.2002 (UNIT I, II, III, IV)
3. R.R. Goldberg, Methods of real Analysis, Oxford & IBH Publishing co. Pvt. Ltd, New Delhi,2019 (UNIT I, II, III, IV)
4. R.G. Bartle and D.R. Sherbert, Introduction to Real Analysis, Wiley India Pvt. Ltd, Fourth Edition, 2016 (UNIT I, II, III, IV)
5. D.Somasundaram and B Choudhary, First Course in Mathematical Analysis, Narosa publishing house New, Delhi, Eighth Reprint 2013 (UNIT I, II, III, IV)
6. P.K. Jain and S.K. Kaushik, An Introduction to Real Analysis, S. Chand & Company Ltd. New Delhi, First Edition 2000 (UNIT I, II, III, IV)
7. Shanti Narayan and M.D. Raisinghania, Elements of Real Analysis, S. Chand & Company Ltd. New Delhi, Fifteenth Revised Edition 2014 (UNIT I, II, III, IV)
8. Shanti Narayan and P.K. Mittal, A course of Mathematical Analysis, S. Chand & Company Ltd. New Delhi, Reprint 2016

Course – VIII BMT402 **Algebra II**

Course Objectives: student will be able to…

1. Develop the skills to use various groups and to prove various results.
2. Present the relationship between abstract algebraic structures with familiar group theory.
3. Study Subgroups, Normal subgroup, Cyclic Subgroups, Homomorphism and Permutation Group
4. Present the concept of Kernel of Homomorphism and Permutation Group structure

Credits (Total Credits 2)	**SEMESTER-IV BMT402 Algebra II**	**No. of hours per unit/credits**
UNIT - I	**Subgroups**	**(12)**
	1.1 Subgroups 1.1.1 **Definition:** Subgroups with illustrations **1.2 Results on Subgroups** 1.2.1 **Theorem:** A non empty subset H of a group G is a subgroup of G If and only if (i) $a, b \in H \Rightarrow ab \in H$ (ii) $a \in H \Rightarrow a^{-1} \in H$ 1.2.2 **Theorem:** A non empty subset of a group G is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$ 1.2.3 **Theorem:** A non empty finite subset H of a group G is a subgroup of G iff H is Closed under multiplication. **1.3 Centre of a Group** 1.3.1 **Definition:** Centre of a Group, Normalizer of an element with illustration. 1.3.2 **Theorem:** Centre of group G is subgroup of group G. 1.3.3 **Theorem:** Normalizer of an element group G is subgroup of group G. **1.4 Cosets** 1.4.1 **Definition:** Coset and examples 1.4.2 **Theorem:** Let H be a subgroup of G then i) $Ha = H \Leftrightarrow a \in H$ *and* $aH = H \Leftrightarrow a \in H$ ii) $Ha = Hb \Leftrightarrow ab^{-1} \in H$ and $aH = bH \Leftrightarrow a^{-1}b \in H$ iii) $Ha(aH)$ is a subgroup of G iff $a \in H$ 1.4.3 **Theorem:** $Ha = \{x \in G \mid x \equiv a \bmod H\} = cl(a)$ for any a in G **1.5 Lagrange's Theorem** 1.5.1 Theorem: If G is a finite group and H is a subgroup of G then $o(H)$ divides $o(G)$.	

	1.6 Index of a subgroup 1.6.1 **Definition:** Index of subgroup H in G with illustration **1.7 Theorem: For** subgroups H and K of G, HK is a subgroup of G iff HK=KH	
UNIT - II	**Cyclic Groups**	**(11)**
	2.1 Cyclic groups 2.1.1 **Definition:** Cyclic group, generator of a cyclic group 2.1.2 Examples on 2.1.1 **2.2 Results on Cyclic groups** 2.2.1 **Theorem:** Order of a cyclic group is equal to the order of its generator. 2.2.2 **Theorem:** A subgroup of cyclic group is cyclic. 2.2.3 **Theorem:** Every cyclic group is abelian. 2.2.4 **Theorem:** If G is finite group then order of any element of G divides order of G. 2.2.5 **Theorem:** An infinite cyclic group has precisely two generators. **2.3 Euler ϕ function** 2.3.1 **Definition:** Euler's $\emptyset$ function 2.3.2 **Theorem:** Number of generators of a finite cyclic group of order n is $\emptyset(n)$. **2.4 Euler and Fermat's Theorem** 2.4.1 **Euler's Theorem:** Let $a, n(n \geq 1)$ be any integers such that $\gcd(a, n) = 1$ then $a^{\emptyset(n)} \equiv 1(\mathrm{mod} n)$ 2.4.2 **Fermat's Theorem:** For any integer a and prime p $a^p \equiv a(\mathrm{mod} p)$ 2.4.3 Examples on 2.4.1 and 2.4.3	
UNIT - III	**Normal groups**	**(11)**
	3.1 Normal groups 3.1.1 Definitions: Normal subgroups, Simple group 3.1.2 Examples **3.2 Results on Normal Groups** 3.2.1 **Theorem:** A subgroup H of group G is normal in G iff $g^{-1}Hg = H, g \in G$. 3.2.2 **Theorem: A** subgroup H of group G is normal in G iff $g^{-1}hg \in H$ for all $h \in H, g \in G$. 3.2.3 **Theorem:** A subgroup H of group G is normal in G iff the product of two right (left) cosets of H in G is again a right (left) coset of H in G.	

	3.3 Quotient groups 3.3.1 **Definition: Quotient groups with illustration.** 3.3.2 **Theorem:** If G is finite group and N is normal subgroup of G then $o\left(\frac{G}{N}\right) = \frac{o(G)}{o(N)}$. 3.3.2 **Theorem:** Every quotient group of cyclic group is cyclic.	
UNIT - IV	**Homomorphism, Permutation Group**	**(11)**
	4.1 Homomorphism 4.1.2 Definitions: Homomorphism, Epimorphism, Monomorphism, Endomorphism and Automorphism. 4.1.2 Examples on 4.4.1 4.1.3 **Theorem:** If $f: G \to G'$ is homomorphism then (i) $f(e) = e'$ (ii) $f(x^{-1}) = [f(x)]^{-1}$ (iii) $f(x^n) = [f(x)]^n$, nis an integer. **4.2 Kernel of Homomorphism** 4.2.1 **Definition:** Kernel of Homomorphism with illustration 4.2.2 **Theorem:** If $f: G \to G'$ is homomorphism then ker f is a normal subgroup of G. 4.2.3 **Theorem:** A homomorphism $f: G \to G'$ is one-one if and only if $\ker f = \{e\}$. **4.3 Isomorphism Theorems** 4.3.1 **Fundamental Theorem of group Homomorphism:** If $f: G \to G'$ is an onto homomorphism with $K = \ker f$ then $\frac{G}{K} \cong G'$. 4.3.2 **Second theorem of Isomorphism :(Statement only)** Let H and K be two subgroups of group G, where H is normal in G, then $\frac{HK}{H} \cong \frac{K}{H \cap K}$. 4.3.3 **Third theorem of Isomorphism (Statement only):** Let H and K be two normal subgroups of group G, such that $H \subseteq K$ then $\frac{G}{K} \cong \frac{G/H}{K/H}$. **4.4 Permutation Group** 4.4.1 **Cayley Theorem:** Every group G is isomorphic to a permutation group. 4.4.2 **Theorem:** Set of even permutations is a normal subgroup of S_n Alternating group.	

Course outcomes: Student should be able to…

1. Explain subgroups and its results to provide numerous examples.
2. Apply Eulers and Fermats theorem.
3. Illustrate quotient groups with the help of normal subgroups.
4. Derive homomorphism and isomorphism theorems.

References:

1. V.K. Khanna and S. K. Bhambri, A course in abstract Algebra, Vikas Publishing house Private Limited, New Delhi, Fifth Edition 2016 (Unit I, II, III, IV)
2. J.B. Fraleigh, A first course in abstract Algebra, Narosa Publishing House New Delhi, Tenth Reprint 2003 (Unit I, II, III, IV)
3. A.R. Vasishtha, Modern Algebra, Krishna Prakashan, Meerut 1994 (Unit I, II, III, IV)
4. M.Artin, Algebra, Prentice Hall of India, New delhi,1994 (Unit I, II, III, IV)
5. I.N. Herstein, Topics in Algebra, Wiley India Pvt. Ltd,1975 (Unit I, II, III, IV)

BMP403- Mathematics Practical –IV

Course Objectives: Student will be able to…

1. Understand groups, sequences and series to study their properties.
2. Study double integration over given region, polar coordinates, change of variables etc.
3. Learn use of functions in C programming.
4. Use programming to solve mathematical problems with Numerical methods.

Credits (Total Credit 02)	**SEMESTER-IV BMP-403 Mathematics Practical –IV**	**No. of hours per unit/credits**
	Group A	
1	Examples on Cyclic Group	
2	Examples on Normal Subgroup	
3	Permutation Group	
4	Homomorphism and Group	
5	Comparison test and Cauchy’s Root test	
6	D’Alembert’s Ratio test and P-test	

7 8	Double Integration over the given region Double Integration: Change of order of integration	
9	Double Integration: Change of co-ordinate axis	
10	Double Integration by using Polar Co-ordinates	
	Group B	
1	Function	
2	Trapezoidal Rule and its Program	
3	Simpsons (1/3) rd rule and program	
4	Simpsons (3/8) th rule and program	
5	Gauss Elimination Method	
6	Gauss Jordan Method	
7	Gauss-Seidel Method	
8	Euler's Method	
9	Euler's Modified Method	
10	Runge-Kutta second & fourth order Method	

Course Outcomes-Students should be able to…

1. Apply various tests to check the convergence of series of real numbers.
2. Simplify double integration with change of order and change of coordinates.
3. Evaluate mathematical problems with the help of Numerical methods and C programming
4. Design programmes to solve system of equations and differential equations.

Practical References:

1. Shantinarayan, A Text Book of Matrices, S. Chand Co. Pvt. Ltd. Raminagar, New Delhi,2010.
2. David. M. Burton, Elementary Number Theory ,7th Edition.2017 McGraw Hill Education.
3. Schaum’s Outline, Discrete Mathematics, (3rd Edition): Seymour hipschutz, Marehipson, Tata MaGraw-Hill Publishing Company Ltd., New Delhi,2010.
4. V.K. Khanna and S. K. Bhambri, A course in abstract Algebra, Vikas Publishing house Private Limited, New Delhi, Fifth Edition 2016
5. J. B. Fraleigh, A first course in abstract Algebra, Narosa Publishing House New Delhi, Tenth Reprint 2003
6. A. R. Vasishtha, Modern Algebra, Krishna Prakashan, Meerut 1994
7. M. Artin, Algebra, Prentice Hall of India, New delhi,1994
8. I. N. Herstein, Topics in Algebra, Wiley India Pvt. Ltd,1975.