Rayat Shikshan Sanstha's

YASHAVANTRAO CHAVAN INSTITUTE OF SCIENCE, SATARA

(AUTONOMOUS)

Lead college of

Karmaveer Bhaurao Patil University, Satara

Syllabus For

Master of Science

Part - II

MATHEMATICS

Syllabus to be implemented w.e.f. June 2024

as Per NEP-2020

Preamble:

The Master of Science program in Mathematics, Second Year, aims to provide advanced knowledge and expertise in various branches of mathematics. Building upon the foundation laid in the first year, this curriculum delves deeper into theoretical frameworks, computational methodologies, and their applications across diverse fields.

Through a combination of rigorous theoretical studies, problem-solving exercises, and practical applications, students will cultivate a profound understanding of advanced mathematical concepts. Emphasis is placed on developing critical thinking, analytical reasoning, and mathematical modelling skills necessary for addressing complex real-world challenges.

This syllabus is designed to offer a comprehensive exploration of key topics in mathematics. Students will engage with cutting-edge research, contemporary developments, and interdisciplinary connections, preparing them for both academic pursuits and professional endeavours.

The syllabus is prepared after discussion at length with number of faculty members of the subject and experts from industries and research fields. The units of the syllabus are well defined, taking into consideration the level and capacity of students

Credit Framework for M.Sc. II

Structure of Course: M.Sc. – II Semester – III

Level	Semester	Course Code	Course Title	No. of Lectures Per Week	Credits
		Discipline Specific Courses (Mandatory)			
6.5	III	MMT 531	Field Theory	4	4
		MMT 532	Integral Equations	4	4
		MMT 533	Number Theory	4	4
		Discipline Specific Elective (Choose Any one among two)			
		MMT 534 E-I MMT 534 E-II	E-I) Fuzzy Mathematics E-II) Operations Research	2	2
		MMP 535	Research Project	12	6
		MMP 536	LAB- III (based on MMT-531, 532 and 533)	4	2
Total					**22**

Structure of Course: M.Sc. – II Semester –IV

Level	Semester	Course Code	Course Title	No. of Lectures Per Week	Credits
		Discipline Specific Courses (Mandatory)			
6.5	IV	MMT 541	Functional Analysis	4	4
		MMT 542	Advanced Discrete Mathematics	4	4
		MMT 543	Algebraic Number Theory	4	4
		Discipline Specific Elective (Choose Any one among two)			
		MMT 54 E-I MMT 544 E-II	E-I) Fractional Differential Equations E-II) Fuzzy Relations	4	4
		MMP 545	On Job Training (OJT).	8	4
		MMP 546	LAB- IV (based on MMT-541, 542 and 543)	4	2
Total					**22**

SEMESTER III

MMT 531 Field Theory

Course Objectives: Student should be able to…

1. understand Galois theory of polynomial equations.
2. study the structure of finite field.
3. learn the formula for general polynomial of degree five or higher.
4. acquire the knowledge of computations in specific examples of finite fields.

Credits= 4	**SEMESTER-III** **MMT 531: Field Theory**	**No. of hours per unit**
UNIT-I	**Algebraic Extensions of Fields**	**15**
	Extension of a field, Degree of extension, Embedding of fields, Algebraic extensions, Finitely generated extension, Algebraically closed fields	
UNIT-II	**Normal and Separable Extension**	**15**
	Splitting field, Normal extension, Multiple roots, Prime field, Separable extension, Perfect field	
UNIT-III	**Galois Theory**	**15**
	Automorphism groups and fixed fields, Fundamental theorem of Galois Theory, Galois extension, Fundamental Theorem of Algebra,	
UNIT-IV	**Application of Galois Theory to Classical Problems**	**15**
	Cyclotomic polynomial, Cyclic extension, Crossed homomorphism	

Course Outcomes: Student will be able to…

1. illustrate the basis and degree of a field over its subfield.
2. determine splitting field for the given polynomial over the given field.
3. illustrate primitive nth roots of unity and nth cyclotomic polynomial.
4. create constructions with straight edge and compass with the help of Galois theory.

Reference Books:

1. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, **Basic Abstract Algebra**, 2nd edition, Cambridge University Press, UK. (Asian edition) 2005. (Unit I, II, III, IV)
2. Nathan Jacobson, **Basic Algebra I**, 2nd Edition, W. H. Freeman and company, New York. (Unit I, II, III, IV)
3. I. N. Herstein, **Topics in Algebra**, 2nd Edition Reprint, Wiley India Pvt.Ltd,2006. (Unit I, II, III, IV)
4. U. M. Swamy, A. V. S. N. Murthy, **Algebra: Abstract and Modern**, Pearson Education, 2012. (Unit I, II, III, IV)
5. John B. Fraleigh, **A first course in Abstract Algebra**, 6th Edition, Narosa Publishing House, New Delhi. (Unit I, II, III, IV)
6. I. T. Adamson, **Introduction to Field Theory**, 2nd Edition, Cambridge University Press, 1982. (Unit I, II, III, IV)
7. M. Artin, **Algebra**, PHI, 1996. (Unit I, II, III, IV)
8. Ian Stewart, **Galois Theory**,4th Edition, CRC Publication,2015. (Unit I, II, III, IV)

MMT 532 Integral Equations

Course Objectives: Student should be able to…

1. understand fundamental mathematical ideas and techniques that lie atthe core of integral equation approach of problem solving.
2. study numerical solutions of integral equations as well as on solving elliptical boundary value problems.
3. study the solutions of Fredholm integral equations and its variousmethods.
4. understand Applications of Laplace and Fourier transforms tosolutions of Volterra integral equations.

Credits= 4	**SEMESTER-III** **MMT 532 Integral Equations**	**No. of hours per unit**
UNIT I	**Linear integral Equations**	**15**
	Classification of linear integral equations, Conversion of initial value problem to Volterra integral equation, Conversion of boundary value problem to Fredholm integral equation, Separable kernel, Fredholm integral equation with separable kernel, Fredholmalternative, Homogeneous Fredholm Equations and Eigen functions.	
UNIT II	**Solutions to Linear integral Equations**	**15**
	Solutions of Fredholm integral equations by: Successive approximations Method, Successive substitution Method, Adomain decomposition method, Modified decomposition method, Resolvent kernel of Fredholm equations and its properties, Solutions of Volterra integral Equations, Successive approximations method, Neumann series, Successive substitution Method.	
UNIT III	**Applications of Laplace and Fourier transforms**	**15**
	Solution of Volterra integral equations by Adomian decomposition method, and the Modified decomposition method, Resolvent kernel of Volterra equations and its properties, Convolution type kernels, Applications of Laplace and Fourier transforms to solutions of Volterra integral equations, Symmetric Kernels**:** Fundamentalproperties of eigenvalues and eigenfunctions for Symmetric kernels, expansion in eigenfunctions and bilinear form.	

UNIT IV	**Hilbert Schmidt Theorem and its consequences**	**15**
	Hilbert Schmidt Theorem and its consequences, Solution of symmetric integral equations, Operator method in the theory of integral equations, Solution of Volterra and Fredholm integrodifferential equations by Adomian decomposition method, Green's function: Definition, Construction of Green's function and its use in solving boundary value problems.	

Course Outcomes: Student will be able to…

1. differentiate Fredholm and Volterra integral equations.
2. solve Fredholm and Volterra integral equations with various methods.
3. analyze the applications of Laplace and Fourier transforms to solutions ofintegral equations.
4. evaluate Volterra and Fredholm integrodifferential equations by Adomiandecomposition method.

Reference Books:

1. R. P. Kanwal, **Linear Integral Equation- Theory and Technique**, Academic Press, 1971 (Unit I, II, III, IV).
2. Abdul-Majid Wazwaz, **Linear and Nonlinear Integral Equations- Methods and Applications**. Springer, 2011 (Unit I, II, III, IV).
3. L. G. Chambers, **Integral Equations- A Short Course**, International Text Book Company, 1976 (Unit I, II, III, IV).
4. M. A. Krasnov, et.al. **Problems and exercises in Integral equations**, Mir Publishers, 1971 (Unit I, II, III, IV).
5. C. D. Green, **Integral Equation Methods**, Thomas Nelson and Sons, 1969. (Unit I, II, III, IV).
6. J. A. Cochran, **The Analysis of Linear Integral Equations**, McGraw Hill Publications, 1972 (Unit I, II, III, IV).

MMT 533 Number Theory

Course Objectives: Student should be able to…

1. learn the fundamental definitions and concepts related to divisibility theory in integers.
2. study the properties of congruences and their role in solving number theory problems.
3. understand the properties of various number theoretic functions.
4. acquire the knowledge of applications of number theory concepts in cryptography or other fields.

Credits= 4	**SEMESTER-III** **MMT 533 Number Theory**	**No. of hours per unit**
UNIT I	**Divisibility Theory in the Integers**	**15**
	Review of divisibility, Division algorithm, G.C.D., Euclid's lemma, Euclidean algorithm, Diophantine equation $ax + by = c$. Fundamental theorem of Arithmetic, Euclidean number, Goldbach Conjecture, Dirichlet's theorem.	
UNIT II	**Theory of Congruences**	**15**
	Basic properties of Congruence, Special divisibility tests, Linear Congruences, Chinese Remainder Theorem, Fermat's theorem, Fermat's factorization method, Wilson's theorem,	
UNIT III	**Number Theoretic Functions**	**15**
	Number Theoretic functions: The functions τ and σ, Multiplicative function, Mobius Inversion formula, Greatest integer function, An Application to the Calendar, Euler's phi function, Euler's theorem, properties of phi function Euler's Generalization of Fermat's theorem: Euler's phi function, Euler's theorem, properties of phi function.	
UNIT IV	**Primitive Roots and Indices**	**15**
	Order of an integer modulo n, Primitive roots, Primitive roots for primes, composite numbers having primitive roots, Theory of Indices, Quadratic reciprocity law, the Legendre symbol and its properties, Quadratic reciprocity, quadratic reciprocity with composite moduli.	

Course Outcomes: Student will be able to…

1. define basic terms related to divisibility theory, congruences, number theoretic functions, primitive roots, and indices.
2. explain the concepts of divisibility, congruences, and modular arithmetic using examples.
3. solve linear congruences and systems of congruences using appropriate techniques.
4. formulate conjectures based on observed patterns in divisibility, congruences, and number theoretic functions.

Reference Books:

1. D. M. Burton, **Elementary Number Theory**, 7th Edition, Universal book stall, New Delhi,2015 (Unit I, II, III, IV).
2. S. B. Malik, **Basic Number Theory,** 2nd Revised Edition, Vikas Publishing House, 2005 (Unit I, II, III, IV).
3. George E. Andrews, **Number Theory**, Hindustan Pub. Corp, 1972 (Unit I, II, III, IV).
4. I. Niven, H. S. Zuckerman, H. L. Montgomery, **An Introduction to Theory of Numbers**, 5th Edition, John Wiley & Sons,1991 (Unit I, II, III, IV).
5. S. G. Telang, M. Nadkarni, J. Dani, **Number Theory**, Tata McGraw Hill Publishing Co. New Delhi, 2001. (Unit I, II, III, IV).

MMT 534: E-I Fuzzy Mathematics

Course Objectives: Student should be able to…

1. identify with the basic structure of fuzzy sets and classical sets.
2. explore Decomposition theorems, Extension principle of fuzzy sets.
3. learn the need of fuzzy sets, fuzzy relations and its applications.
4. emphasize the appropriate fuzzy theory corresponding to uncertain and imprecise collected data.

Credits= 2	**SEMESTER-III** **MMT 534: E-I Fuzzy Mathematics**	**No. of hours per unit**
UNIT I	**Fuzzy Sets**	**08**
	Definition: Fuzzy sets and crisp sets, Examples of fuzzy sets, Standard operations, α-cuts, Properties of α-cuts, Support, height, normality & sub-normality of fuzzy set, Core of fuzzy set, cardinality, Level cuts.	
UNIT II	**Representation of Fuzzy sets**	**07**
	Representation of Fuzzy sets, Decomposition theorems, Extension principle.	
UNIT III	**Operations on fuzzy sets**	**08**
	Fuzzy complement, Equilibrium and dual point, increasing & decreasing generator function, Fuzzy intersection: t-norms, Fuzzy union: t-conorms.	
UNIT IV	**Fuzzy numbers**	**07**
	Fuzzy numbers, Arithmetic operations on Intervals, Arithmetic operations on fuzzy numbers.	

Course Outcomes: Student will be able to…

1. apply structure of fuzzy sets and its properties.
2. analyze extension principle to find direct images of fuzzy set.
3. examine operations on fuzzy sets.
4. evaluate arithmetic operations on fuzzy numbers.

Reference Books:

1. George J. Klir, Bo Yuan, **Fuzzy sets and Fuzzy Logic Theory and Applications**, PHI.Ltd.2000. (Unit I, II, III, IV)
2. M. Grabish, Sugeno, Murofushi, **Fuzzy Measures and Integrals Theory and Applications**, PHI, 1999. (Unit I, II, III, IV)
3. H.J. Zimmerermann, **Fuzzy Set Theory and its Applications**, Kluwer, 1984. (Unit I, II, III, IV)
4. M. Hanss, **Applied Fuzzy Arithmetic, An Introduction with Engineering Applications**, Springer- Verlag Berlin Heidelberg 2005. (Unit I, II, III, IV)
5. M. Ganesh, **Introduction to Fuzzy Sets & Fuzzy Logic**, PHI Learning Private Limited, New Delhi 2006. (Unit I, II, III, IV)
6. Timothy J. Ross, **Fuzzy Logic with Engineering Applications**, 3rd Edition, John Wiley and Sons, 2011. (Unit I, II, III, IV)

MMT 534: E-II Operations Research

Course Objectives: Student should be able to…

1. understand complex mathematical models in management science, and transportation problem.
2. study advanced methods for large-scale transportation.
3. gain some applications of graph theory to practical problems and other branches of Mathematics.
4. learn Kuhn Tucker, Wolfe's method and Beale's method for solving Non-linear programming.

Credits=2	**SEMESTER-III** **MMT 534: E-II Operations Research**	**No. of hours per unit**
UNIT I	**Convex sets and their properties**	**08**
	Convex sets and their properties. Lines and hyper planes convex set Important Theorems, polyhedral convex set convex combination of vectors, convex hull, convex polyhedron, convex cone, simplex and convex function, General formulation of linear programming Matrix form of LP Problem, definitions of standard LPP, Fundamental Theorem of linear programming. Simplex method, computational procedure of simplex method, problem of degeneracy and method to Resolve degeneracy.	
UNIT II	**Linear Programming**	**07**
	Revised simplex method in standard form I, Duality in linear programming duality theorem, Integer linear programming, Gomory's cutting plane method, Branch and Bound and linear Programming.	
UNIT III	**Dynamic Programming**	**08**
	Dynamic programming. Bellman's principle of optimality, solution of problems with a finite number of stages. Application of dynamic programming in production, inventory control.	
UNIT IV	**Non-linear programming**	**07**
	Non-linear programming unconstrained problems of maximum and minimum Lagrangian method Kuhn Tucker necessary and sufficient conditions, Wolfe's method, Beale's method. Directed graphs: Definition, Indegree and outdegree, Tournaments, traffic flow. Networks: Flows and Cuts, The Ford and Fulkerson Algorithm, Separating seen.	

Course Outcomes: Student will be able to…

1. understand linear programming (LP) models for shortest path, maximum flow, minimal spanning tree, critical path.
2. use some solution methods for solving the nonlinear and linear optimization problems.
3. analyze the general nonlinear programing models.
4. derive the Khun-Tucker optimality conditions.

Reference Books:

1. S.D. Sharma, Himanshu Sharma, **Operations Research Theory, Methods and Applications**, Kedar Nath Ram Noth,2010. (Unit I, II, III, IV)
2. Kanti Swarup, P.K. Gupta and Manmohan, **Operations Research**, S. Chand & Sons, New Delhi 2001. (Unit I, II, III, IV)
3. Hamady. A. Taha, **Operations Research**, 10th Edition, Pearson 2017. (Unit I, II, III, IV)
4. P. K. Gupta, D. S. Hira, **Operations Research,** 7th Edition, S. Chand Publication 1976. (Unit I, II, III, IV)

MMP 535: Research Project

Credits =6	Semester III MMP 535: Research Project	
	Students will undertake research in specific area of his Major/Core with an advisory supported by a teacher/Faculty member. Students are required to take 6 credit Research Project for semester III under the guidance of faculty members.	

MMP 536: Lab III – Partial Differential Equations

Course Objectives: Student should be able to…

1. understand classification of partial differential equations and its solutions.
2. learn partial differential equations of first order with various methods.
3. acquire the knowledge of second order partial differential equations and its applications.
4. study various boundary value problems and its solutions.

Credits= 2	**SEMESTER-III** **MMP 536: Lab III-Partial Differential Equations**	**No. of hours per practical**
1	Formation of first order partial differential equations by eliminating arbitrary constants.	3
2	Construction of first order partial differential equations by eliminating arbitrary function.	3
3	Classification of first order partial differential equations.	3
4	Finding the general integral of quasi linear partial differential equations of first order.	3
5	Finding the complete integral, singular integral and particular integral of first order partial differential equations.	3
6	Solution of Pfaffian differential equations in three variables.	3
7	Checking the compatibility of partial differential equations of first order and finding the common solution.	3
8	Complete integral of first order partial differential equations using Charpit's method.	3
9	Complete integral of first order partial differential equations using Jacobi's method.	3
10	Integral surface to a given curve for a linear partial differential equation.	3
11	Integral surface of a non-linear partial differential equations for given curve.	3
12	Integral surface of quasi linear partial differential equation through a given curve by method of characteristics.	3
13	Classification of second order partial differential equations by change of independent variable.	3
14	D'Alembert's solution of one-dimensional wave equation which	3

	describes the vibration of an infinite string with both ends are not fixed.	
15	Solution of partial differential equation of second order obtained by vibration of a string of finite length using method of separation of variables.	3
16	Solution of Heat conduction problem by using method of separation of variables.	3
17	Solution of Heat conduction problem by using Duhamel's principle.	3
18	Finding the potential function corresponding to equipotential surfaces.	3
19	Solution of Laplace equation by using method of separation of variables.	3
20	Solution of Laplace equation by using Fourier transform.	3

Course Outcomes: Student will be able to…

1. understand first order partial differential equations and its classifications.
2. use various methods to solve partial differential equations of first order.
3. analyze second order partial differential equations and its applications.
4. evaluate boundary value problems.

Reference Books:

1. T. Amaranth, **An elementary course in Partial differential equations**, Narosa Publication,1987.
2. Fritz John, **Partial Differential Equations**, 4th Edition, Springer Science & Business Media, 1991.
3. I.N. Sneddon, **Elements of Partial Differential Equations**, Dover Publication 2013.

SEMESTER IV

MMT 541: Functional Analysis

Course Objectives: Student should be able to…

1. study of the main properties of bounded operators between Banach and Hilbert space.
2. study basic result associated to different types of converges in normedspaces.
3. understand Banach and Hilbert spaces and self-adjoint operators.
4. understand use contractions of Banach spaces.

Credits= 4	**SEMESTER-IV** **MMT 541: Functional Analysis**	**No. of hours per unit**
UNIT I	**Normed Linear Spaces**	**15**
	Normed linear spaces, Banach spaces, Quotient spaces, Continuouslinear transformations, Equivalent norms, Finite dimensional normed spaces and properties, Conjugate space and Separability, The Hahn-Banach theorem and its consequences.	
UNIT II	**The Open mapping theorem and uniform boundedness principle**	**15**
	Second conjugate space, the natural embedding of the normed linear space in its second Conjugate space, Reflexivity of normed spaces, Weak * topology on the conjugate space. The open mapping theorem, Projection on Banach space, the closed graph theorem, the conjugate of an operator, the uniform boundedness principle.	
UNIT III	**Hilbert Spaces**	**15**
	Hilbert spaces: examples and elementary properties, Orthogonalcomplements, The projection theorem, Orthogonal sets, The Bessel’s inequality, Fourier expansion and Parseval’s equation, separable Hilbert spaces, The conjugate of Hilbert space, Riesz's theorem, The adjoint of an operator	
UNIT IV	**Self-Adjoint operators**	**15**
	Self-adjoint operators, Normal and Unitary operators, Projections, Eigen values and eigenvectors of an operator on a Hilbert space, The determinants and spectrum of an operator, The spectral theorem on a finite dimensional Hilbert space.	

Course Outcomes: Student will be able to…

1. understand the fundamental properties of normed spaces and transformations between them.
2. use specific techniques for bounded operators over normed spaces.
3. apply the notion of dot product and Hilbert spaces to solve problems on its properties.
4. derive the spectral theorem to the resolution of integral equations.

Reference Books:

1. G. F. Simmons, **Introduction to Topology and Modern Analysis**, Tata McGraw Hill, 1963. (Unit I, II, III, IV)
2. Erwin Kreyszig, **Introductory Functional Analysis with Applications**, John Wiley and Sons, 1978. (Unit I, II, III, IV)
3. G. Bachman and L. Narici, **Functional Analysis**, Academic Press, 1972. (UnitI, II, III, IV)
4. A. E. Taylor, **Introduction to Functional analysis**, John Wiley and sons,1958. (Unit I, II, III, IV)
5. J. B. Convey, **A course in Functional Analysis**, Springer-Verlag, 1985. (UnitI, II, III, IV)
6. B. V. Limaye, **Functioned Analysis**, New age international, 1996. (Unit I, II,III, IV)

MMT 542: Advanced Discrete Mathematics

Course Objectives: Student should be able to…

1. learn key concepts in graph theory.
2. understand the relationship between matrices and graphs, and interpret matrix representations of graphs.
3. study properties of lattices and Boolean algebraic structures.
4. gain the knowledge of Pigeonhole Principle to solve complex combinatorial problems.

Credits= 4	**SEMESTER-IV** **MMT 542: Advanced Discrete Mathematics**	**No. of hours per unit**
UNIT I	**Graph theory**	**15**
	Graph Theory: Definition, examples and properties, Simple graph, Graph isomorphism, Bipartite graphs, Complete Bipartite graph, regular graph, sub-graphs, spanning sub-graph, Edge deleted subgraph, Vertex deleted sub-graph, Union and intersection of two graphs, complements of a graph, self-complementary graph, paths and cycles in a graph, Eccentricity, Radius and diameter of a connected graph, Peterson graph, Wheel graph, First theorem of graph theory.	
UNIT II	**Matrix representation of graph and Pigeonhole principle**	**15**
	The Matrix representation of a graph, Adjacency matrix and Incidence matrix of a graph, Definition and properties of a tree, bridges, spanning trees, Inclusion exclusion principle, Examples on Inclusion exclusion principle, Pigeonhole principle, Examples on Pigeonhole principle.	
UNIT III	**Discrete numeric functions and Linear recurrence relations**	**15**
	Discrete numeric function, sum and product of two numeric functions, generating functions, Linear recurrence relations with constant coefficients, Particular solutions of Linear recurrence relations, Total solutions.	
UNIT IV	**Lattices and Boolean Algebra**	**15**
	Ordered sets, Hasse diagrams of posets, Supremum and infimum, Isomorphic ordered sets, well-ordered sets, Lattices, Bounded lattices, Distributive lattices, Complements complemented lattices, Boolean algebra, Basic definitions, Basic theorems, duality, Boolean algebras as lattices	

Course Outcomes: Student should be able to…

1. demonstrate understanding of fundamental concepts in graph theory.
2. explain the significance of graph theory in various fields including computer science, operations research, and social network analysis.
3. analyze properties of graphs and evaluate their connectivity, planarity, and coloring.
4. design new algorithms or improve existing ones based on advanced graph theory concepts to address complex problems.

Reference Books:

1. Seymon Lipschutz and Mark Lipson, **Discrete Mathematics,** 2^{nd} Edition, Tata McGraw Hill Publishing Company Ltd. New Delhi. (Unit I, II, III, IV)
2. John Clark and Derek Holton, **A first book at Graph Theory**, Applied Publishers Ltd. (Unit I, II, III, IV)
3. S. G. Telang, M. Nadkarni, J. Dani, **Number Theory**, Tata McGraw-Hill Publishing Co. New Delhi, 2001. (Unit I, II, III, IV)

MMT 543: Algebraic Number Theory

Course Objectives: Student should be able to…

1. understand algebraic numbers and algebraic integers and find its integral basis.
2. study the existence of factorization and norms.
3. learn the relationship between factorization of numbers and of ideals.
4. gain the class number and fitness of class group.

Credits= 4	**SEMESTER-IV** **MMT 543: Algebraic Number Theory**	**No. of hours per unit**
UNIT I	**Field Extensions and Free Abelian Groups**	**15**
	Revision of rings, polynomial rings and fields, Field extensions, Symmetric polynomials, Modules, Free Abelian groups.	
UNIT II	**Ring of integers**	**15**
	Algebraic Numbers, Algebraic number fields, Conjugates and Discriminants, Algebraic integers, Integral Bases, Norms and Traces, Ring of integers, Quadratic fields, Cyclotomic fields.	
UNIT III	**Prime Factorization**	**15**
	Factorization into irreducible, Noetherian rings, Dedekind rings, Examples of Non- Unique factorization into irreducible, Prime factorization, Euclidean Domains, Euclidean quadratic fields.	
UNIT IV	**Class groups and Class numbers**	**15**
	Ideals, Prime factorization of ideals, Norm of an ideal, Nonunique factorization in cyclotomic fields, Two-squares theorem, Four-squares theorem, class groups and class numbers, Finiteness of the Class groups.	

Course Outcomes: Student will be able to…

1. identify properties of number fields.
2. understand the arithmetic of algebraic number fields.
3. use theorem about integral bases, and about unique factorization in to ideals.
4. analyze class numbers and find the relationship between factorization of numbers as well as of ideals.

Reference Books:

1. Stewart and D. Tall, **Algebraic Number Theory and Fermat's last theorem**, 3rd Edition, 2002. (Unit I, II, III, IV)
2. N. Jacobson, **Basic Algebra - I**, 2nd Edition, Dover Publications,2012. (Unit I, II, III, IV)
3. Murty, M. Ram, Esmonde, Jody Indigo, **Problems in Algebraic Number Theory**, Springer 2008. (Unit I, II, III, IV)
4. J. Neukirch, **Algebraic Number Theory**, Grundlehren der mathematischen Wissenschaften, 1999. (Unit I, II, III, IV)

MMT 544 E-I: Fractional Differential Equations

Course Objectives: Student should be able to…

1. study types of Fractional Differential Equations.
2. understand Mellin transforms of fractional derivatives-Mellin transforms of the Riemann-Liouville fractional integrals and fractional derivative.
3. learn the solution of FDE by using Laplace transform method.
4. gain mathematical models using fractional derivatives to solve application problems such as harmonic oscillators and circuits.

Credits= 4	**SEMESTER-IV** **MMT 544 E-I: Fractional Differential Equations**	**No. of hours per unit**
UNIT-I	**Grünwald-Letnikov (GL) Fractional Derivative**	**15**
	Brief review of Special Functions of the Fractional Calculus: Gamma Function, Mittag-LefflerFunction, Fractional Derivative and Integrals: Grünwald-Letnikov (GL) Fractional Derivatives-Unification of integer order derivatives and integrals, GL Derivatives of arbitrary order, GL fractional derivative of $(t-a)^{\beta}$, Composition of GL derivative with integer order derivatives, Composition of two GL derivatives of different orders.	
UNIT-II	**Riemann- Liouville (RL) Fractional Derivative**	**15**
	Riemann- Liouville (RL) fractional derivatives- Unification of integer order derivatives and integrals, Integrals of arbitrary order, RL derivatives of arbitrary order, RL fractional derivative of $(t-a)^{\beta}$, Composition of RL derivative with integer order derivatives and fractional derivatives, Link of RL derivative to GL approach.	
UNIT-III	**Caputo's Fractional Derivative**	**15**
	Caputo's fractional derivative, generalized functions approach, Left and right fractional derivatives. Properties of fractional derivatives: Linearity, The Leibnitz rule for fractional derivatives, Fractional derivative for composite function, Riemann-Liouville fractional differentiation of an integral depending on a parameter,Behavior near the lower terminal, Behavior far from the lower terminal.	
UNIT-IV	**Laplace Transform, Fourier Transform and Mellin Transform**	**15**
	Laplace transforms of fractional derivatives- Laplace transform of the Riemann- Liouville fractional derivative, Caputo derivative and Grünwald-Letnikov fractional derivative. Fourier transforms of fractional integrals and derivatives. Mellin transforms of fractional derivatives-Mellin transforms of the Riemann-Liouville fractional integrals and fractional derivative; Mellin transforms of Caputo derivative.	

Course Outcomes: Student will be able to…

1. understand the concept of fractional derivatives and its forms.
2. apply properties like Linearity, The Leibnitz rule for fractional derivatives.
3. simplify Mellin transforms of fractional derivatives-Mellin transforms of the Riemann-Liouville fractional integrals and fractional derivative
4. evaluate fractional differential equations using various methods.

Reference Books:

1. Igor Podlubny, **Fractional Differential Equations,** San Diego, Academic Press 1999. (Unit I, II, III, IV)
2. A. Kilbas, H.M. Srivastava, J.J. Trujillo, **Theory and Applications of Fractional Differential Equations**, Elsevier, Amsterdam, 2006. (Unit I, II, III, IV)
3. Kai Diethelm, **The Analysis of Fractional Differential Equations**, Springer, 2010. (Unit I, II, III, IV)
4. L. Debnath, D. Bhatta, **Integral Transforms and Their Applications**, CRC Press, 2010. (Unit I, II, III, IV)

MMT 544 E-II: Fuzzy Relations

Course Objectives: Student should be able to…

1. understand Fuzzy relations in different forms.
2. study Fuzzy relational equations and methods of solutions.
3. learn Fuzzy qualified and quantified propositions.
4. gain Fuzzy approximate reasoning in different forms.

Credits= 4	**SEMESTER-IV** **MMT 544 E-II: Fuzzy Relations**	**No. of hours per unit**
UNIT I	**Fuzzy Relations**	**15**
	Projections and cylindrical Extensions Binary Fuzzy Relations on single set, Fuzzy equivalence relations, Fuzzy Compatibility Relations, Fuzzy ordering Relations Fuzzy morphisms Sup-i Compositions and inf-w_i compositions.	
UNIT II	**Fuzzy Relational Equations**	**15**
	Fuzzy Relation Equation, Problem Partitioning, solution methods, Fuzzy relational equations based on sup-I and inf-wi compositions, Approximate solutions.	
UNIT III	**Fuzzy Propositions**	**15**
	Fuzzy propositions, Fuzzy Quantifiers, Linguistic Hedges, Inference from conditional fuzzy propositions, Qualified and quantified propositions.	
UNIT IV	**Fuzzy Approximate Reasoning**	**15**
	Approximate Reasoning: -Fuzzy expert systems, Fuzzy implications, selection of Fuzzy implications, Multi-conditional Approximate Reasoning, Role of fuzzy relational equations, Interval valued Approximate Reasoning.	

Course Outcomes: Student will be able to…

1. understand fuzzy relations in different forms.
2. apply various methods of solutions for solving Fuzzy relational equations.
3. classify Fuzzy Propositions into qualified and quantified propositions.
4. design Fuzzy expert system using approximate reasoning.

Reference Books:

1. George J Klir, Bo Yuan, **Fuzzy sets and Fuzzy Logic Theory and applications**, PHI, Ltd.2000. (Unit I, II, III, IV)
2. M.Grabish, Sugeno, and Murofushi, **Fuzzy Measures and Integrals: Theory and Applications**, PHI, 1999. (Unit I, II, III, IV)
3. M. Ganesh, **Introduction to Fuzzy Sets & Fuzzy Logic,** PHI Learning Private Limited, New Delhi. (Unit I, II, III, IV)

MMP 545: On Job Training (OJT)

Credits =4	Semester III MMP 545: On Job Training (OJT)	
	OJT will provide the opportunities for internship with local/regional industries, business organization, health and allied areas, local government, etc. so that students may actively engaged with the employability opportunities. Students will undergo 4 credit work-based learning/OJT/internship.	

MMP 546: Lab IV- Combinatorics

Course Objectives: Student should be able to…

1. learn applications of permutation, combination and pigeonhole principle.
2. explore inclusion exclusion principle, Derangements and combinatorial proofs.
3. identify Rook polynomial and recurrence relations.
4. emphasize group theory and Polya's enumeration theorem.

Credits =2	**SEMESTER-IV MMP 546: Lab IV- Combinatorics**	**No. of hours per practical**
1	Applications of the sum rule and the product rule.	3
2	Applications of permutations and combinations of n-distinct objects.	3
3	Combinatory proof of identities.	3
4	Applications of the Pigeonhole Principle.	3
5	Applications of generalized permutations and combinations	3
6	Use of Multinomial theorem to find expansion of mathematical expression.	3
7	Applications of Inclusion–Exclusion Principle to find cardinalities of finite sets.	3
8	Applications of Derangements of n-distinct objects.	3
9	To find Ordinary and Exponential Generating Functions of given sequence.	3
10	Solution of Linear Recurrence Relations with Constant Coefficients.	3
11	Applications of Fibonacci Sequence.	3
12	Applications of Generating Function Models.	3
13	Computation of Rook Polynomials of given chess boards.	3
14	Applications of Partition of Numbers.	3
15	Applications of Stirling Number of First Kind.	3
16	Applications of Stirling Number of Second Kind.	3
17	Applications of System of Distinct Representatives.	3
18	Use of Menger, Hall, Konig – Equivalence.	3
19	Computation of Cycle Index of a Group.	3
20	Applications of Polya's Enumeration Theory.	3

Course Outcomes: Student will be able to…

1. apply the combinatorial identities and pigeonhole principle.
2. analyze derangements of objects and cardinality of sets using inclusion exclusion principle.
3. evaluate recurrence relations using ordinary and exponential generating functions.
4. derive the System of Distinct Representatives and Polya's Enumeration theorem.

Reference Books:

1. Richard A. Brualdi, **Introductory Combinatorics**, 5th Edition, Pearson, 2009.
2. Alan Tucker, **Applied Combinatorics**, 6th Edition, Wiley, 2012.
3. Mitchel T. Keller and Willian T. Trotter, **Applied Combinatorics**, 2017th Edition.
4. V.K. Balakrishnan, **Schum's Outline of Theory and Problems of Combinatorics**, Mc. Grew Hill Education, 1994.
5. Douglas B. West, **Combinatorial Mathematics**, Cambridge University Press, 2020.